MA 748 - Fourier Analysis, section # 48086 Spring 2025

September 18, 2024

- **Instructor:** Professor Atanas Stefanov,
- **Office:** UH 4049, Phone: (205) 934-8551.
- **Class meetings:** MWF 12:20 - 1:10, UH 4004
- **Office Hours:** TBD.
- **Email:** stefanov@uab.edu
- **Prerequisite**: MA 645, MA 648 and MA 655 (which may be taken concurrently) or permission of the instructor.
- **Text:** Fundamentals of Fourier Analysis by L. Grafakos.
  I will also post a copy of the book on CANVAS.
- **Topics:**
  - L^p spaces, weak L^p spaces, basic convolution inequalities (Hausdorf-Young's).
  - Real and complex interpolation (Marcinkiewich and Riesz-Thorin theorems), applications.
  - Fourier Transform - properties, inversion formula. Applications (Green's formula for Poisson's equation, Sobolev embedding).
  - Singular integrals – examples (Hilbert, Riesz transforms). Singular integrals - Intro to Calderon-Zygmund theory and multipliers (Hörmander - Mikhlin's theorem).
  - Littlewood-Paley theory, fractional order Sobolev spaces. Product estimates (Leibnitz rule). Applications.
  - Semigroups and generators. Abstract general theorem for local well-posedness of semi-linear PDE's.

  - Method of stationary phase, Strichartz estimates. Applications: the heat equation, the Schrödinger equation, in particular local well-posedness for semilinear Schrödinger equation.
  - If time permits: Local well-posedness for Navier-Stokes in all dimensions, global well-posedness for large data in 2D Navier-Stokes[1].

- **Grades:** Your grade for this course will be determined by the number of points that you accumulate from the homework assignments. I will offer two HW projects, due during the semester, plus a final project, due in the Final's week.

[1]The 3D problem, global well-posedness for large data in the 3D Navier-Stokes problem, is one of the most famous unsolved "millennial" open problems, with an award of $\$1,000,000$.