Numerical Linear Algebra
MA 660-1F, Spring 2025

Instructor: Dr. Y. Zeng, UH 4012

Time & Location: MWF 13:25 - 14:15, UH 4002

Office Hours: Monday, Wednesday 14:30 –15:30 (or by appointment)

Text: A set of class notes (evolved from courses taught by several faculty members in the department) will be provided. These notes contain all definitions, theorems, and examples, but no proofs (which will be presented in detail in class).

References:

- G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 3rd Ed., 1996
- D. S. Watkins, *Fundamentals of Matrix Computations*, Wiley, John & Sons., 2nd Ed., 2002
- L. N. Trefethen and D. Bau, III, *Numerical Linear Algebra*, SIAM, 1997
- R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge Univ Press, 2nd Ed., 2013

Course Contents: Norms and inner products; orthogonal vectors; orthogonal/unitary matrices; adjoint and self-adjoint matrices; bilinear forms and positive definite matrices; Cholesky factorization; machine arithmetic; sensitivity and stability; over-determined linear systems; Schur decomposition and Singular value decomposition; eigenvalues and eigenvectors – sensitivity and computation.

Grading Policy:

Homework assignments	40 %
Midterm exam (Friday, Feb. 28, tentative)	20 %
Final exam (Friday, May 2, 10:45 AM – 1:15 PM)	40 %

Homework Assignments: Homework will be assigned weekly on Monday and due the following Monday, unless announced otherwise. Software package MATLAB may be used in some assignments. Homework will NOT be accepted late. However, the two lowest homework grades will be dropped to account for any missed assignments due to illness or any other circumstance. I am not planning on accepting any excuses except in extraordinary circumstances.

Exams: Midterm and Final exams will be closed-book, in-class and comprehensive.

Preparation for Joint Program Exam: This course covers the material for the numerical linear algebra in the Joint Program Exam. Past exams can be downloaded at

http://www.uab.edu/cas/mathematics/graduate/phd/qualifying-exams-testbank

Problems from past exams will also be used in homework assignments.

Learning Outcomes: By the end of the course, students will be able to do the following.

1. Understand the concepts of vector norms, distance and matrix norms induced by vector norms, and apply their properties in different situations.
2. Understand the concepts of real and complex inner products and be able to apply Cauchy-Schwarz-Buniakowsky inequality.
3. Understand the connection between an inner product and a vector norm.
4. Understand the concepts of orthogonal vectors and orthogonal/orthonormal basis, be able to apply Pythagoras theorem and perform Fourier expansion.
5. Find orthogonal projection of a vector on another, the angle between two vectors and expansion of a vector along an orthonormal basis.
6. Perform Gram-Schmidt orthogonalization to convert a basis into an orthonormal basis.
7. Be able to apply Parceval's identity and Bessel's inequality.
8. Understand the concept of isometry.
9. Understand the concepts of orthogonal operators and unitary operators, and apply their properties.
10. Understand the concepts of adjoint and self-adjoint operators, unitary/orthogonal equivalence, and unitary/orthogonal equivalence of a Hermitean/symmetric matrix to a diagonal matrix.
11. Learn the concepts of bilinear forms, Hermitean and symmetric forms and their properties; be able to link a selfadjoint operator to a Hermitean/symmetric form.
12. Learn the concept of positive definite matrices, their properties and their spectral decompositions.
13. Learn Sylvesters theorem and perform Cholesky factorization for a symmetric, positive definite matrix.

14. Learn topics related to machine arithmetic, such as binary numbers, floating point representation, machine floating point numbers, errors, computational errors and error estimation.

15. Learn topics related to computational sensitivity and stability, condition number of a matrix, round-off error analysis and Wilkinsons analysis for the LU decomposition.

16. Solve overdetermined linear systems by least squares fi; perform QR decomposition by reflections and by rotations.

17. Perform Schur decomposition and singular value decomposition.

18. Learn Rayleigh quotient of a matrix, Courant-Fisher minimax theorem and Bauer-Fike theorem.

19. Understand the concept of condition number of an eigenvalue; learn Gershgorin theorems.

20. Find eigenvalues and eigenvectors by iteration algorithms.