Name:

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FINAL EXAM A
MA227

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Open Book. No notes, etc. No calculators.
CIRCLE YOUR ANSWER. You must show your work and justify your answers to receive credit.

1. Suppose a projectile is fired at a 45 degree angle from the top of a building that is 50 feet tall with an initial speed of v_0 ft./sec.
   (a) Write a position vector that describes the motion of the projectile for all time t.
   (b) If the projectile is 10 feet above the ground $5\sqrt{2}$ seconds later, what was the initial speed?
2. The radius of a right circular cone is increasing at a rate of $\frac{3}{2}$ in/sec while its height is decreasing at a rate of $\frac{2}{3}$ in/sec. At what rate is the volume of the cone changing when the radius is 9 inches and the height is 12 inches?
3. Find the directional derivative of $f(x,y) = \sqrt{xy}$ at $P(3, 27)$ in the direction of $Q(15, 36)$.
4. The dimensions, length, width, and height, of a closed rectangular box[1] are measured to be approximately 40 cm, 30 cm, and 50 cm, respectively, with the same error in each measurement. If you wish to know the surface area of the box within a maximum error of 120 cm^2, what is the maximum error that you can tolerate in measuring the length, width, and height?
5. Let D be the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$, which includes the boundary. Find the absolute maximum and absolute minimum values of $f(x,y) = 13 + xy - x - 2y$ on the set D.
6. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 25$.
7. Evaluate

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{0}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2} dz dx dy.$$

8. (a) Show that the vector field $\mathbf{F}(x,y) = y\mathbf{i} + (x + 4y)\mathbf{j}$ is conservative and find a function $f(x,y)$ such that $\mathbf{F}(x,y) = \nabla f(x,y)$. (b) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

   where C is the upper semicircle that starts at $(0, 5)$ and ends at $(9, 5)$.
9. Use Green's Theorem to find the work done by the force $\mathbf{F}(x,y) = 2x(2x + y)\mathbf{i} + 2xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to $(1, 0)$, then along the (straight) line segment to $(0, 1)$, and then back to the origin along the y-axis.

Date: August 25, 2006.
[1]Being closed it must have a top.