EXAM II

MA 125 CT, CALCULUS I

March 3, 2016

Name (Print last name first): .....

Show all your work, simplify and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $f(x) = \left(x^3 + x\right)^5$.

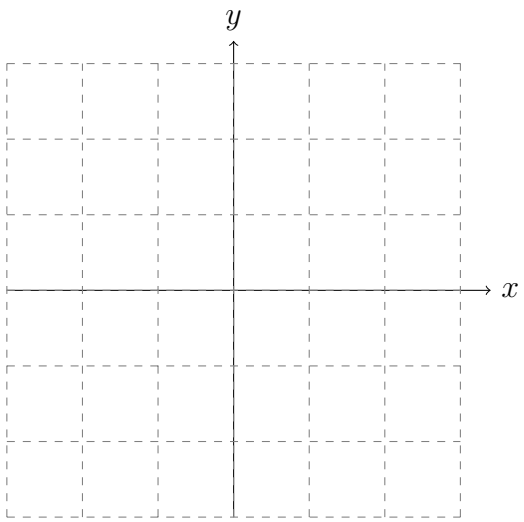
2. Find all critical numbers of the function $f(x) = (2x + 1)^3(1 - x)^7$.

3. Find the absolute maximum and minimum of the function $f(x) = x^4 - 2x^2$ on the interval $[-2, 2]$.

4. Verify that the conditions of the Mean Value Theorem hold. Next find the number c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = x^5$ on the interval $[0, 1]$.

5. Show that the equation $f(x) = x^7 + x^3 + x + 5 = 0$ has exactly one solution. Hint: first show that there is at least one solution. Next show that two distinct solutions is impossible.

6. Suppose that the **derivative** of a function $f(x)$ is given by the graph below.



(a) Use the graph to read off all critical numbers and which of them is the x-coordinate of a local max or a local min of the function $f(x)$.

(b) At which x is the function $f(x)$ most rapidly increasing?

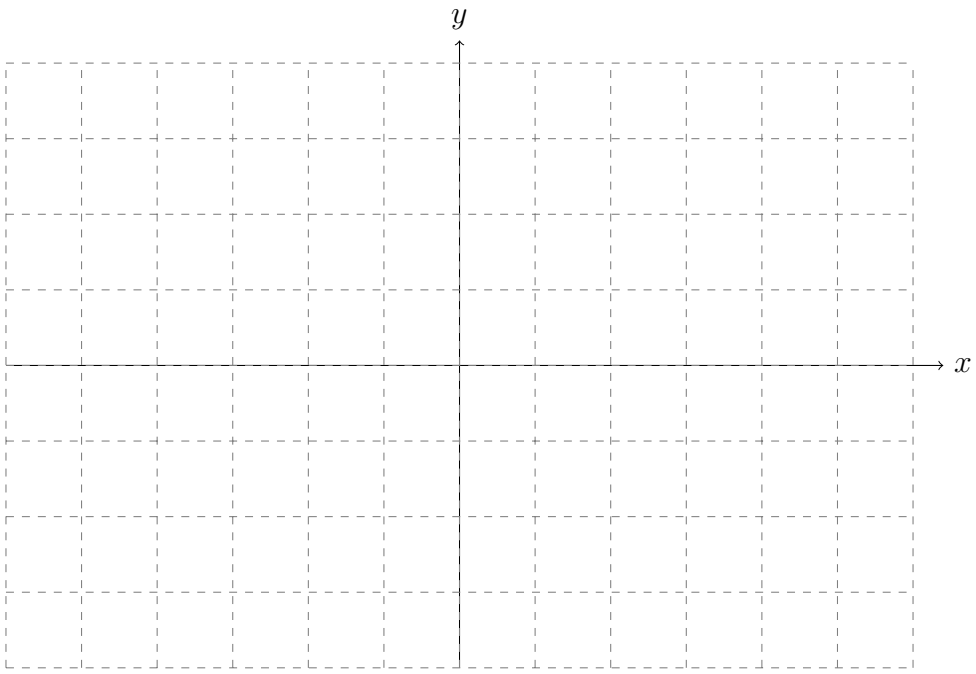
PART II

7. [**10 points**] Suppose you are asked to design a rectangular box of volume 1 m^3 of minimal surface area if one side of the base is twice as long as the second side of the base. State the dimensions of the box of minimal surface area.
   Hint: draw a box with dimensions of the base x and $2x$, and height y so that its volume $V = 2x^2y$. Then compute the area of all sides (including top and bottom); add these and minimize this function.

8. [**20 points**] **Use calculus** to graph the function $f(x) = \dfrac{1+x^2}{1-x^2}$. Indicate

   - x and y intercepts,
   - vertical and horizontal asymptotes (if any),
   - in/de-creasing; local/absolute max/min (if any).

   You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).



9. This question has two parts.

   (a) [**6 points**] Find the linearization of $f(x) = \sqrt[3]{x}$ at $a = 8$

   (b) [**4 points**] Use this linearization to find the approximate value of $\sqrt[3]{8.01}$.