EGR 265, Math Tools for Engineering Problem Solving

April 8, 2015, 50 minutes

TEST III

Name: ....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8*	
Total	

Problem 1 (7+7 points)

(a) Find the gradient of the function $f(x, y) = 3xy^2 + x^3$.

(b) Find the directional derivative of $f(x, y)$ at the point $(1, 1)$ in the direction of the vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$.

Problem 2 (10+4+4 points)

(a) For the function $g(x, y) = e^{x+2y}$ find the rate of steepest ascent at the origin $(0, 0)$.

(b) Find a unit vector in the direction of steepest *ascent* for $g(x, y)$ at the origin.

(c) Find a unit vector in the direction of steepest *descent* for $g(x, y)$ at the origin.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the level survace $xy^2z^3 = 1$ at the point $(1, 1, 1)$.

(b) Also, find parametric equations for the normal line of $xy^2z^3 = 1$ at $(1, 1, 1)$.

Problem 4 (12 points)

Evaluate $\int_C (y-1)\, ds$, where C is the quarter circle or radius 1, centered at the origin, starting at $(1, 0)$ and ending at $(0, 1)$

Problem 5 (14 points)

Find the work done by the force field

$$F(x, y) = y\,\mathbf{i} - x\,\mathbf{j}$$

along the curve parameterized by $x = t^2$, $y = t^3$, $0 \le t \le 1$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = (3x^2 - y)\mathbf{i} + (x + 4y)\mathbf{j}$

(b) $F(x, y) = (3x^2 + y))\mathbf{i} + (x - 4y)\mathbf{j}$

Problem 7 (14 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve parameterized by $x = t$, $y = t \sin t$, $0 \le t \le \pi/2$.

Problem 8* (6 points bonus)

The height of a mountain is described by the function $z = xy - 2x^2 - y^2 + 2y$. Find the x and y coordinates of the top of the mountain.