EGR 265, Math Tools for Engineering Problem Solving
March 9, 2015, 50 minutes

Name: .....

TEST II

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 6y' + 9y = 0, \quad y(0) = 1, \quad y'(0) = 4.$$

Problem 2 (20 points)

Solve the initial value problem

$$y'' + y' - 2y = 5, \quad y(0) = 0, \quad y'(0) = 0.$$

Problem 3 (20 points)

Find the general solution of

$$y'' + 4y = \cos x + \sin x.$$

Problem 4 (20 points)

A mass of 5 kilograms stretches an undamped spring by 25 centimeters. For simplicity, use the approximate value $g = 10$ m/sec^2 for this problem.

(a) Find the value of the spring constant k using its correct metric unit.

(b) Find the angular frequency ω of free oscillations of the spring/mass-system.

(c) Find the equation of motion if the mass is released from the equilibrium position at an upward velocity of 50 cm/sec. Assume here that the positive x-direction is oriented downwards.

(d) Find the first time $t > 0$ at which the mass returns to the equilibrium position.

Problem 5 (8 points)

Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient β kg/sec. How should β be chosen to achieve critical damping?

Problem 6 (12 points)

(a) Find the partial derivatives f_x and f_y of the function $f(x, y) = x^3y^3 + x^2 + y^2$

(b) Find the second order partial derivative g_{xy} of $g(x, y) = e^{x^2+y^2}$

Problem 7* (5 points bonus)

Suppose you want to find a particular solution for $y'' + 2y' + y = 2e^{-x}$ by the Method of Undetermined Coefficients. What is the correct guess you should start from?
(Note that in this problem you don't have to completely solve the DE. Just go as far as needed to make the correct guess and justify this.)