MA 125 - 6C, CALCULUS I
April 14, 2014

Name (Print last name first): .....

Student Signature: .....

TEST III

No calculators are allowed!

PART I

Part I consists of 8 questions, 7 points each. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find all the critical numbers of the function $f(x) = \frac{1}{3}x^3 - 9x$.

Question 2

Find the limit

$$\lim_{x \to 0} \frac{e^x - 1}{\sin(x)}.$$

Question 3

Find the absolute maximum value of the function $g(x) = 4x - x^2$ on the closed interval $[0, 1]$.

Question 4

Find the open interval(s) on which the function $g(x) = x^3 - 27x - 15$ is increasing.

Question 5

Find the open interval(s) on which the function $g(x) = x^3 - 27x - 15$ is concave down.

Question 6

If $f'(x) = (x+2)^2(x-1)^3(x-2)^3$. Find all points of the local maximum and local minimum of the function f. **NOTICE:** the first derivative of the function is already given!

Question 7

If $f''(x) = (x+1)^2(x-1)^3(x-2)^4$. Find all inflection points of the function f. **NOTICE:** the second derivative of the function is already given!

Question 8

Find all vertical and horizontal asymptotes of the function

$$f(x) = \frac{x^2+1}{1-x^2}.$$

PART II

Each problem is worth 14 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Suppose that the derivative of a function $f(x)$ is given by

$$f'(x) = (x-1)^4(x+1)^3(x-2)^2.$$

NOTICE: the derivative of the function is already given! Answer all the following questions.

(a) Find all the critical numbers of the function $f(x)$.

(b) On what interval(s) is the function $f(x)$ increasing? (Justify your answer!)

(c) On what interval(s) is the function $f(x)$ decreasing? (Justify your answer!)

Problem 2

Consider the function

$$f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2 + 1.$$

Answer all the following questions.

(a) Find the (open) interval(s) of increase, and all the (open) interval(s) of decrease.

(b) Find all local maximum and minimum points. [Be sure to give the x and y-coordinates of each point.]

(c) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up.

(d) Find the inflection points. [Be sure to give the x and the y coordinate!]

(e) Use the information from parts (a)–(d) to sketch the graph.

Problem 3

Consider the function

$$f(x) = xe^x.$$

Answer all the following questions.

(a) Find the (open) interval(s) of increase, and all the (open) interval(s) of decrease.

(b) Find all local maximum and minimum points. [Be sure to give the x and y-coordinates of each point.]

(c) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up. Find the inflection points. [Be sure to give the x and the y coordinate!]

(d) Find behavior of the function as $x \to \pm\infty$.

(e) Use the information from parts (a)–(d) to sketch the graph.

Problem 4

(a) Find the limit

$$\lim_{x\to 0}(1+x)^{\frac{1}{\sin(2x)}}.$$

(b) Find the dimensions of a rectangle with area 25 cm^2 whose perimeter is as small as possible. (Show your work!)

Scratch paper