EGR 265, Math Tools for Engineering Problem Solving
March 10, 2014, 50 minutes

Name: .....

TEST II

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 2y' + 5y = 0, \quad y(0) = -2, \quad y'(0) = 2.$$

Problem 2 (20 points)

Find the general solution of

$$y'' + y' - 2y = x + 2.$$

Problem 3 (20 points)

Find the general solution of

$$y'' + 6y' + 9y = 6e^{-x}.$$

Problem 4 (20 points)

A mass of 5 kilograms stretches an undamped spring by 49 centimeters. (Use the value $g = 9.8$ m/sec^2 for this problem.)
(a) Find the value of the spring constant k using its correct metric unit.
(b) Find the angular frequency ω of free oscillations of the spring/mass-system.
(c) Find the equation of motion if the mass is released from rest at a position 20 centimeters below the equilibrium. Assume here that the positive x-direction is oriented downwards.
(d) Find the first time $t > 0$ at which the mass passes through the equilibrium position. (Hint: If your answer to (c) is correct, this can be done without first writing the equation of motion in alternative form.)

Problem 5 (10 points)

Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 50$ kg/sec. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Find the largest interval centered around $x = 1/2$ in which the initial value problem

$$y'' + \frac{1}{x}y' + \frac{1}{x-1}y = 0, \quad y(1/2) = 1, y'(1/2) = 2$$

has a unique solution. Do not try to solve the DE! Give a theoretical justification instead.

Problem 7* (5 points bonus)

Suppose you want to find a particular solution for $y'' - 2y' + y = 2e^x$ by the Method of Undetermined Coefficients. What is the correct guess you should start from?
(Note that in this problem you don't have to completely solve the DE. Just go as far as needed to make the correct guess and justify this.)