Instructor:_____ Name:_____

Final Exam

Calculus I; Spring 2014

Part I

Part I consists of 10 questions, each worth 5 points. You must simplify your answer when possible. You must also show all your work for each of the problems listed.

In 1-4, find y' if:

(1) $y = x^5 \sin(x)$

(2) $y = \frac{x^3+1}{e^x}$

(3) $y = [\cos(x)]^5$

(4) $y = \ln(x^5 + x)$

(5) Find the critical numbers of $y = f(x) = (x-1)^3(x+1)^3$.

(6) Find all local/absolute maxima/minima of the function $y = x^6 - 2x^3$. Make sure to state both x and y values.

(7) Find all asymptotes of the function $y = \frac{x^2+4}{x^3-x}$

(8) Find all x-values where $y = xe^x$ is **increasing**.

(9) Suppose one side of a rectangle is increasing at a rate of $3\,m/s$ while the second side is decreasing at a rate of $2\,m/s$. Find the rate of change of the area of the rectangle when the first side is $5\,m$ and the second side is $7\,m$.

(10) Evaluate the following limit $\lim_{x\to\infty} \dfrac{x}{\ln(x)}$

(11) Use calculus to find the dimensions of a rectangle whose area is 7 and whose perimeter is minimal.

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) **[3 pts]** Evaluate the limit $\lim_{x\to\infty} x\ln(x)$

(2) **[6 pts]** Evaluate the limit $\lim_{x\to 0^+} x\ln(x)$

(3) **[8 pts]** Use linearization at an appropriate point $x = a$ to estimate the value of the function $\sqrt[3]{8.1}$

(4) Note that you are not asked to determine where the function is concave up/down nor do you need to find the points of inflection. **Be careful when computing f′(x)!**

Given the function $y = f(x) = \dfrac{x^2 - 9}{x^2 - 4}$

(a) **[2 pts]** Find the x and y intercepts of the function.

(b) **[2 points]** Find all asymptotes.

(c) **[2 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) **[2 pts]** Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinates of each local max/min).

(e) **[5 pts]** Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **absolute/local maxima/minima**.

(5) **[10 pts]** If $y = f(x) = (x^2 - 1)^2$, find the absolute maximum and minimum of $f(x)$ on the closed interval $[-1, 2]$. (Include the appropriate y values, simplify when possible.)

(6) **[10 pts]** A street light is mounted at the top of a $9\,m$-tall pole. A man $3\,m$ tall walks away from the pole at a speed of $1\,m/s$. How fast is the tip of his shadow moving when he is $15\,m$ from the pole? [You may not need all information in the problem.]