MA 227, CALCULUS III

Spring, 2013

Name (Print last name first): .....

Student Signature: .....

TEST II

10 questions, 10 points each.
SHOW ALL YOUR WORK! CIRCLE YOUR ANSWER!

Question 1

Find the gradient of the function $f(x, y) = xe^{-x^2y}$ at the point $(1, 0)$.

Question 2

Find the directional derivative of the function $f(x, y, z) = x^2z + yz^2$ in the direction of the vector $\vec{v} = \vec{i} + \vec{j} + 2\vec{k}$ at the point $(1, 2, -1)$.

Question 3

Find local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 - 2xy + 2y^2 + y + 2.$$

Question 4

Let $z = x^2y - \frac{x}{y}$. Find equation of the tangent plane at point $(-1, 1)$.

Question 5

Find linear approximation for the function

$$f(x, y) = x^3 - xy^2 + yx^2$$

near point $(1, 2)$.

Question 6

Let $f(x,y) = xy - \frac{y}{x^2}$ and $x = s - t, \quad y = s - t^2$. Find partial derivatives $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

Question 7

Let $f(x,y) = x\cos(y) - x^3 y$. Find all second partial derivatives: $f''_{xx},\ f''_{xy},\ f''_{yy}$.

Question 8

Find equation of the tangent plane to the surface $x^2 + y^3 - z^4 = 4$ at the point $(-2, 1, 1)$.

Question 9

Find the maximum rate of change of $f(x, y) = xy^{1/3} - \frac{y}{x}$ at the point $(-1, 1)$. In which direction does it occur?

Question 10

Find the absolute maximum and absolute minimum of the function $f(x,y) = -x^2 - y^2 + 4y + 1$ on the region $-1 \leq x \leq 0, \quad 0 \leq y \leq 1$. Be sure to provide coordinates of the points and the values of absolute maximum and minimum.