Instructor:_____ Name:_____

Final Exam

Calculus I; Spring 2013

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

In 1-6, find y' if:

(1) $y = x\sin(x)$

(2) $y = \frac{x}{\sin(x)}$

(3) $y = (x^2 + x)^{30}$

(4) $y = f(x) = \ln(x^2 + x)$

(5) $y = x \arcsin(x)$

(6) Find y' (in terms of x and y) if $x^4 + y^4 = xy$

(7) Find the most general form of the **anti**-derivative of $y = f(x) = x^2(x + 1)$

(8) Find the the critical number(s) of the function $y = f(x) = xe^{-x}$

(9) Evaluate $\lim_{x \to \infty} x^2 e^{-x}$

(10) Find the absolute maximum and minimum of the function $y = f(x) = (x+1)^2(x-1)^2$ on $[-1, 2]$. Provide both the x and y values.

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps and justify your answer to earn credit. Simplify your answer when possible.

(1) **[10 pts]** Use implicit differentiation to find the derivative y' if $x^2 + y^2 = \sin(xy)$

(2) **[5 pts]** Find the linearization of the function $y = f(x) = \sqrt{x}$ at $a = 2$.

(3) **[3 pts]** Use the linearization in problem 2 to estimate $\sqrt{9.1}$

(4) Given the function $y = f(x) = \frac{x-1}{x^2}$

(a) **[2 pts]** Find the domain and the x and y intercepts of the function.

(b) **[3 pts]** Find the vertical and horizontal asymptotes of the function.

(c) **[2 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) **[2 pts]** Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinate of each of them). **State absolute max/min if any.**

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) **[1 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point when possible).

(g) **[5 pts]** Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **x**,**y**-intercepts, local/absolute maxima, minima and point(s) of inflection.

(5) **[5 pts]** If $y = xe^{-x}$ find the absolute max and min on the interval $-1 \leq x \leq 3$. (Include the appropriate y values of the maximum and minimum.)

(6) **[10 pts]** A 2 m tall person walks toward a street light at a speed of 1 m/s. If the street light is 5 m tall, how fast is the tip of his shadow moving when he is 2m from the light post?

Scratch paper