MA-227/6D: Calculus III
Test#3, April 25, 2012

Time available: 110 min.
Each problem is 15 points

Your name (print):

Your signature:

Please always explain your answer, at least by including your calculations. You should work on this sheet. A right answer without calculation brings you no credit!

1. Calculate

$$\iint_R x^2 y e^{x^3 y} dA,$$

$R = [0,1] \times [0,1]$.

2. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y = z$, $x = 0$, $z = 0$ in the first octant.

3. Evaluate the integral by converting to polar coordinates.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \cos(x^2+y^2)dydx.$$

4. Let the lamina D be bounded by the curves $y = e^x$, $y = 0$, $x = 0$, and $x = 1$ with mass density function $\rho(x, y) = 1$. Find the moment of inertia I_x and the total mass m.

5. Find the volume of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.

6. Evaluate the integral by switching to cylindrical coordinates.

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{1-x^2-y^2} (x^2 + y^2) dz dy dx.$$

7. Calculate

$$\iiint_E z^2 dV,$$

where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

8. *Use the given transformation* to evaluate the integral.

$$\iint_R (x^2 + xy + y^2) dA,$$

where R is the region bounded by the ellipse $x^2 + xy + y^2 = 1$; $x = \sqrt{1/3}u + v$, $y = \sqrt{1/3}u - v$.