Calculus II, Final Exam, Spring 2012

Name:_____

You must show all your work and give reasons for your answers. Good luck!

(1) (5 points) Find the angle between the vectors $< -1, 0, 1 >$ and $< 2, -1, 2 >$. You may express your answer using $\arccos(x)$.

(2) (5 points) Find the equation of the plane containing the point $(-1, 2, 1)$ which is perpendicular to the line

$$\begin{cases} x = 1 + t \\ y = 1 - t \\ z = 1 + 2t \end{cases}$$

(3) (5 points) Evaluate $\int x^2 \cos(x^3)\, dx$.

(4) (5 points) Evaluate $\int \sqrt{x}(x^2 + x)\, dx$.

(5) (5 points) Evaluate $\int \arctan(x)\, dx$.

(6) (5 points) Express $f(x) = \frac{3x}{2+x}$ as a power series. **Also** state the interval of convergence.

(7) (5 points) Use series to approximate $e^{-\frac{1}{10}}$ with an error less than 10^{-6}. [You do not need to compute and add the terms in the sum.]

(8) (5 points) Evaluate $\int \sin^2(x)\,dx$.

(9) (5 points) Find the area of the region bounded by the curves $x = 0$, $x = \pi$, $y = x + 3$ and $y = \sin(x)$.

(10) (5 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 0$, $x = 1$, $y = x^3 + x + 1$ and $y = x - 1$ around the line $x = -2$.

(11) (5 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 0$, $x = 1$, $y = x^3 + x + 1$ and $y = x - 1$ around the line $y = -2$.

(12) (10 points) Find the interval and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{\left(\frac{x}{5}\right)^n}{n}$$

(13) (15 points) Evaluate

$$\int \frac{2}{x^3 + x}\, dx$$

(14) (10 points) Find the work done in pumping all the water out of a container which is a round sphere of radius 5 m and which is half full (i.e. the water has to be lifted to the top of the sphere before it can be removed). [You may use the approximation $g \approx 10\, m/sec^2$ and $\rho = 1000\, kg/m^3$.]

(15) (10 points) Approximate $\int_0^{\frac{1}{10}} \frac{1}{1+x^5}\,dx$ with an error less than 10^{-12}. (You do not need to compute and add the terms in the sum.)