CALCULUS 1, TEST 4

MA 125 NAME_____

100 points/ No calculator Spring, 2012

<u>Part 1.</u> Part 1 consists of 8 questions. Clearly write your answer in the space provided after each question. Show your work and simplify your answer as much as possible. (6 points each.)

1. Find all critical numbers for the function $f(x) = x + 3x^{2/3}$.

2. Determine using calculus the open interval(s) on which the function $f(x) = x^5 - 5x$ is increasing.

3. Use calculus to determine the interval(s) on which the function $f(x) = xe^x$ is concave downward.

4. Find the most general antiderivative F of $f(x) = \dfrac{x^2 - x + 1}{x}$.

5. The function $f(x) = 2x^2$ satisfies the hypotheses of the Mean Value Theorem on the interval [0,2]. Find the number c that satisfies the conclusion of the Mean Value Theorem.

6. Find the inflection points (both x and y-coordinates) of $y = f(x) = 5x^4 - x^5$.

7. Use the Closed Interval Method to find the maximum and minimum values of $f(x) = x^3 - 12x - 1$ on the interval [-5, 4].

8. Suppose an object moves a long a straight line where its acceleration is given by $a = 3\cos t$. Find the velocity v and position s for the object's motion if it is known that $v(0) = 2$ and $s(0) = 4$

<u>PART 2.</u> Part 2 consists of 4 problems worth 13 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.

1. Show how you use calculus to find two positive numbers whose product is 20 and whose sum is a minimum.

2. A rectangle has a perimeter (length around the outside) of 120 ft. Find the dimensions which make the area as large as possible.

3. For the function $f(x) = \dfrac{x^2 - 9}{x^2 - 4}$, determine

a. the domain of f,

b. the x and y intercepts if any .

c. the vertical asymptotes, if any

d. the horizontal asymptotes, if any (show work here to justify your answer)

e. Use symmetry tests to find any symmetry that exists.

4. Suppose f is a function whose derivatives are as follows:

$$f'(x) = \frac{4x+2}{x^{2/3}} \qquad \text{and} \qquad f''(x) = \frac{8(x-2)}{3x^{5/3}}$$

Suppose it is also known that the **domain** of the function is the interval $(-\infty, \infty)$; the **x-intercepts** are (-1, 0) and (0, 0); and that there are **no vertical or horizontal asymptotes** for the graph of f.

a. On what open interval(s) is the function f increasing? _____

On what open interval(s) is the function f decreasing? _____

b. On what open interval(s) is the graph of the function f concave upward? _____

On what open interval(s) is the graph of the function f concave downward? _____

b. Graph the function f, based on the information above, and indicate the x-values of any local maximum or minimum points (you won't know the exact y-values for the max/min points).

SCRATCH PAPER