MA 125 Calculus 1 **TEST 1** Name________________________

100 points/ No calculators Spring, 2012

<u>Part 1</u> Part 1 consists 8 questions worth 5 points each. Clearly indicate your answer in the space provided after each question. ***<u>Show all of your work for full credit!</u>***

In evaluating the limits that appear in some of the problems in Part 1, use the limit theorems, limit properties, and techniques studied thus far (but no educated guessing). Consider ∞ and $-\infty$ as possible values. If a limit has no value, not even ∞ or $-\infty$, state this and indicate why the limit fails to exist.

1. If functions *f*, *g* are continuous with $f(2) = 5$ and $\lim_{x\to 2}[4f(x) - g(x)] = 11$, find $g(2)$.

2. Evaluate $\lim_{x\to 0}\frac{\cos x + 1}{5x - 2}$.

3. Evaluate $\lim_{x\to -2}\frac{x^2 - x - 6}{x + 2}$.

4. If $f(x) = x\sin(x^2)$ + 1 and $g(x) = \sqrt{x}$, what is the function $f \circ g$?

5. Determine the *x*-values where the following function $y = \dfrac{x^2 - 1}{x^2 - 2x - 3}$ **fails** to be continuous.

6. Evaluate $\lim\limits_{x \to \infty} \dfrac{-3x^2 + 2x}{9x^2 - 4x + 1}$

7. Evaluate $\lim\limits_{x \to 2^+} \dfrac{1 - x}{x - 2}$

8. Evaluate $\lim\limits_{h \to 0} \dfrac{(1 + h)^2 - 1}{h}$.

Part 2. **Part 2 consists of 5 problems #1 is worth 12 points, #2-4 are worth 10 each, and #5 is worth 18 points. Again, show all the work required to work the problems.**
A final answer (even if correct) without the relevant steps will not earn full credit.

1. a. Suppose the position s (in feet) of a particle moving along the y-axis at time t (in sec) is given by $s = y = t^2 - 2t$, $t \geq 0$. Find the **average** velocity v_{ave} (complete with units of measure) of the particle over the time interval from $t = 1$ to $t = 4$ seconds.

   b. Find the equation of the tangent line to the function $y = f(x)$ at the point P whose x-coordinate is 2 if it is known that $f(2) = -3$ and $f'(2) = 7$

2 a. For what value of the constant c is the function $f(x) = \begin{cases} x^2 + c & \text{if } x \geq 2 \\ -cx + 8 & \text{if } x < 2 \end{cases}$

continuous at $x = 2$?

b. Does the $\lim_{x \to \infty} \sin x$ have a value? If so, what is the value (include ∞ *and* $-\infty$ as possible values)? If not, why not?

3. Let $f(x)=\dfrac{3}{x}$. Use the ***limit definition of the derivative*** to find the derivative $f'(x)$.

4. Sketch the graph of an example of a function f such that

$\lim_{x\to 0^-} f(x)=1$, $\lim_{x\to 0^+} f(x)=-1$, $f(0)$ is undefined $\lim_{x\to\infty} f(x)=-1$

$\lim_{x\to 2^-} f(x)=0$, $\lim_{x\to 2^+} f(x)=1$ $f(2)=1$,

.

5. a. The graph of $y = f(x)$ is given to the right.

i. What is the domain of f? ______________

ii. For which value (s) of a in the domain of f does the $\lim_{x \to a} f(x)$ **fail** to exist?

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iii. For which value(s) of a in the domain of f does f **fail** to be continuous at $x = a$ **?**

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b. The graph of $y = f(x)$ is given to the right. Which of the following statements here are *true (T)* and which are *false (F)*?

i. $\lim_{x \to 2} f(x)$ does not exist.

ii. The function f is continuous from the right at $x=1$.

iii. The function f is differentiable at $x=0$.