EGR 265, Math Tools for Engineering Problem Solving
February 7, 2011, 50 minutes

Name (Print last name first): .....

Student ID Number: ....

TEST I

Problem 1

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $y' + x = \cos y$

(b) $\dfrac{y' + e^x}{y} = x$

(c) $y^{(4)} - y^5 = x^2$

(d) $y^{(3)} + \dfrac{e^x y}{x} = 1$

Problem 2

(a) Which of the following functions are solutions of $x^4 y' + 2xy^2 = 4x^5$? (8P)

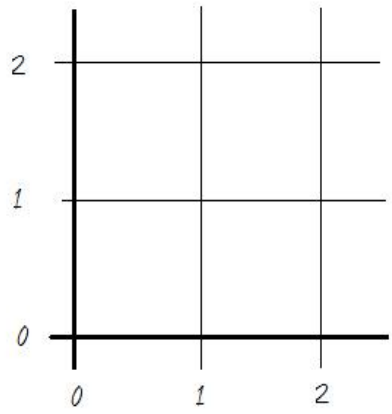
$$y_1 = x^2, \quad y_2 = -x^2, \quad y_3 = x, \quad y_4 = -2x^2.$$

(b) Which of the functions from part (a) solve the initial value problem $x^4y' + 2xy^2 = 4x^5$, $y(0) = 0$? (4P)

(c)* (Bonus) Does your answer to part (b) agree with the content of the Existence and Uniqueness Theorem for first order ODEs? If yes, why? If no, why not? (5P*)

Problem 3

(a) In the 3×3-grid of points $x = 0, 1, 2$ and $y = 0, 1, 2$ provided in the figure below draw a direction field for $y' = x^2(y - 1)$. (8P)



(b) Without solving the DE, use the direction field to guess the solution of the IVP $y' = x^2(y - 1)$, $y(1) = 1$. Check that your guess is correct by verifying that it is a solution of the IVP. (4P)

Problem 4

Solve the IVP (15P)

$$y' = 2xy^2, \quad y(0) = -1.$$

Problem 5

Solve the IVP (15P)

$$y' + y = x, \quad y(0) = 2$$

Problem 6

Solve the IVP (15P)

$$y' - xe^{-y} = 0, \quad y(0) = 0$$

Problem 7

The number of bacteria in a culture is given by $n(t)$, where the time t is measured in hours. An initial population of $n(0) = 100$ bacteria grows at a constant rate $k = n'(t)/n(t)$. After one hour 150 bacteria are present.

(a) Find the rate of growth k by solving the differential equation for $n(t)$. (10P)

(b) Find an expression for the time t_d necessary for the number of bacteria to double: $n(t_d) = 200$. (5P)

Note: Your answers will contain natural logarithms which do not need to be evaluated.

(a)

(b)