Time available: 110 min

Your name (print):

Your signature:

Please always explain your answer, at least by including your calculations. You should work on this sheet. A right answer without calculation brings you no credit!

1. At what point(s) does the curve

$$\vec{r}(t) = \langle t - 2,\ 11t^2 - 11t + 14,\ 3t \rangle$$

intersect the paraboloid $y = 3x^2 + z^2$?

10 points

2. Find a parametric equation for the tangent line to the previous curve at the point $P(0, 36, 6)$.

10 points

3. Find the curvature of the curve $\vec{r}(t) = \cos t\vec{i} + \cos t\vec{j} - 3\sin t\vec{k}$ at the point $P(1, 1, 0)$.

20 points

4. Find the vectors T, N, and B for the curve of problem 3 at the given point.

20 points

5. Find the tangential and normal components of the acceleration vector for the curve $\vec{r}(t) = t\vec{i} + 2t\vec{j} + t^2\vec{k}$ at the generic point $\vec{r}(t)$.

10 points

6. The motion $\vec{r}(t)$ takes place for positive time (always $t > 0$), $\vec{a}(t) = 6t\vec{i} + \frac{1}{t^2}\vec{j} + 6t\vec{k}$, $\vec{v}(1) = 3\vec{i} - \vec{j} + 3\vec{k}$, $\vec{r}(1) = \vec{i} + \vec{k}$. Compute $\vec{r}(t)$.

20 points

7. Find the vectors $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{B}(t)$ for the curve

$$\vec{r}(t) = \langle 3t,\ 4\cos t,\ 4\sin t \rangle.$$

10 points

8. Find the velocity, acceleration, and speed of a particle with the position function

$$\vec{r}(t) = \langle t,\ t\sin t,\ -t\cos t \rangle.$$

10 points

9. Find the curvature of the space curve

$$\vec{r}(t) = t\vec{i} + t\vec{j} - t^2\vec{k}$$

at any point.

10 points

10. Let $r(t) = \langle t, 3t, t^2 \rangle$. Find the tangential and normal components of the acceleration, i. e. find a_T and a_N.

10 points