Calculus II, Exam IV, Spring 2011

Name:_____

Student signature:_____

Show all your work and give reasons for your answers. In case of convergence of series state absolutely convergent, conditionally convergent, or divergent. Good luck!

Part I. All problems in Part I are 7 points

(1) Test the series for convergence (you must justify your answer). If convergent, find the sum. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$.

(2) Test the series for convergence (you must justify your answer). $\sum_{n=2}^{\infty} \frac{n^2+1}{n-1}$.

(3) Test the series for convergence (you must justify your answer). $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(4) Test the series for convergence (you must justify your answer). $\sum_{n=1}^{\infty} \frac{2^n}{n!}$.

(5) Find the power series representation of the function $f(x) = \frac{x}{1-x}$. Also state the interval of convergence!!

(6) Use power series to approximate the value of $\sin(1/10)$ with an error less than 10^{-6}.

(7) Find the power series representation of the function $f(x) = e^{(x^3)}$. Also state the interval of convergence. [You may use the power series for e^x.]

Part II. All problems in part II are 17 points.

(1) Find the interval and radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$

(2) Approximate the value of the integral with an error less than 10^{-10}:

$$\int_0^{(1/10)} \sin(x^3)\, dx$$

.

(3) Find the power series representation of $f(x) = \frac{x^3}{1+x^7}$. Also state the interval and radius of convergence.