**EGR/MA 265, Math Tools for Engineering Problem Solving**
May 03, 2010, 10:45 AM - 1:15 PM

Name (Print Last Name First): .....

Student Signature: .....

**FINAL EXAM**

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$2(1+x)yy' = 1, \quad y(0) = -1.$$

Problem 2 (8 points)

A radioactive isotope has a half-life of 20 years.
(a) Find its decay rate $k$ (which should be a negative number).
(b) If the initial amount of the isotope is 1 gram, how much of it is left after 10 years?
(c) How long does it take for the isotope to decay to 20 percent of its original amount?
Note: Write your answers in terms of natural logarithms, which do not need to be evaluated.

Problem 3 (14 points)

Consider the second order differential equation

$$y'' - y' - 2y = e^{2x}. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 0$, $y'(0) = 0$.

Problem 4 (12 points)

A mass of 10 kg stretches a spring by 100 cm. Include the correct units in all your answers below.
(a) Find the spring constant $k$, assuming that $g = 10$ m/s$^2$.
(b) What is the frequency at which the mass oscillates?
(c) Find the equation of motion of the mass if it is released from rest at a position 50 cm below the equilibrium position (choose the positive $x$-axis to be oriented downward).
(d) Find the first positive time at which the mass passes through the equilibrium position.

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = \ln\left(\sqrt{x^2 + y^2}\right)$.
(b) Evaluate the directional derivative of $f(x, y)$ at the point $P(1, 0)$ in the direction from $P$ to the point $Q(4, 4)$.
(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $(1, 0)$. Also find the rate of increase in this direction.

Problem 6 (8 points)

Determine parametric equations of the normal line to the graph of $z = \dfrac{y}{x+y}$ at the point $(1, -2, 2)$.

Problem 7 (8 points)

Find the line integral

$$\int_C xy^4\,ds,$$

where $C$ is the right half of the unit circle centered at the origin, starting at $(0,-1)$ and ending at $(0,1)$.

Problem 8 (12 points)

(a) Is the force field $F(x, y) = 3x^2y^4\mathbf{i} + 4x^3y^3\mathbf{j}$ conservative? If your answer is yes, then find a potential function $\Phi(x, y)$ for it.
(b) Find the work done by the force field $F$ from part (a) along a portion of the parabola given by the parametric equations $x(t) = t$, $y = t^2$, $0 \leq t \leq 1$.

Problem 9 (10 points)

A lamina of constant density $\rho(x, y) = 1$ is bounded by the curves $x = y^2 - 1$ and $x = 0$.
(a) Find the lamina's mass.
(b) Find the lamina's centroid. Use geometric considerations to simplify your work.

Problem 10 (10 points)

Find the double integral of the function $f(x, y) = x^2 + y^2$ over the region in the second quadrant which is bounded by the circles $r = 2$ and $r = 4$.

SCRATCH PAPER

SCRATCH PAPER