MA 126 CALCULUS II SPRING 2010

FINAL EXAM

Name (Print):………………………………………… Instructor:………………………….

Part 1

Each question is worth 4 points.

Part I consists of 11 questions. Place your answer on the answer-line next to the question. Show your work on each problem. Space is provided between questions.

1 Evaluate $\int x(x^2-5)^4 dx$ ____________________

2. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converges or diverges. ____________________

3. Use a power series to approximate $e^{-1/10}$ with error less than 0.0005. ____________________

4. Evaluate $\int \sin^3 x \cos^2 x \, dx$

5. Determine whether the ***series*** $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1}$ converges or diverges. Show work. __________________

6. Find the equation of the plane through the point $(-1,1,2)$ and perpendicular (normal) to the line with parametric equations $x=1+t,\ y=2-t,\ z=1-2t$. __________________

7. Determine whether the ***sequence*** $\{a_n\} = \left\{ \frac{n^3}{1-2n^3} \right\}$ converges or diverges. __________________

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8. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2}{7^n}$. __________________

9. If $f(x) = \int_2^x \sqrt{t^5 - 5}\, dt$, find $f'(x)$. __________________

10. Determine whether $\sum_{n=0}^{\infty} \frac{(-12)^n}{n!}$ is absolutely convergent, conditionally convergent, or divergent. __________________

11. Evaluate $\int x^2 e^{2x} dx$. __________________

PART II

Each problem is worth 8 points

Part II consists of 7 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

Problem 1

Two planes are given by the equations $x = y + 2z - 2$ and $z = x - 2y + 2$.

(a) Find parametric equations for the line of intersection of these planes.

(b) Determine the cosine of the angle between the two planes.

Problem 2

(a) Find a power series representation for $f(x) = \dfrac{1}{1+x^2}$ in the form $\sum_{n=0}^{\infty} c_n x^n$. On what interval of x-values does the series equal the function? Explain or show work that justifies your choice of interval.

(b) Use your answer from (a) to assist you in finding a power series representation for $g(x) = \ln(1+x^2)$.

Problem 3

Evaluate and simplify.

(a) $\displaystyle\int \frac{1}{y(y^2+1)}\,dy$

(b) $\displaystyle\int_3^5 \frac{x-2}{x-1}\,dx$

Problem 4

Find the area of the region enclosed by the line $y = x - 2$ and the parabola $y^2 = 2x + 4$.

Problem 5

Consider the region R in the xy plane bounded by $y = \sqrt{x}$, $x = 0$, and $y = 2$. Set up an integral, **but do not evaluate it**, for the **volume** of the solid generated by rotating region R about each line given below. In each case your integral should be complete with limits and in terms of just one variable.

(a) $y = 2$

(b) $x = 4$

Problem 6

a. Find the length of the curve $y = \dfrac{2(x^2+1)^{3/2}}{3}$, $0 \le x \le 1$.

b. Find the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$. Remember to check endpoints if they exist.

Problem 7

A tank has the shape of an inverted (vertex down) circular **cone** with height 12 meters and radius 5 meters. It is filled with water to a height of 9 meters. Find the work required to empty the tank by pumping all the water to the top of the tank. (The density of water is 1000 kg/m^3.) Note. It is sufficient to set up an integral representing the work, but **you do not have to evaluate the integral**. Your work integral should be in terms of just one variable and have appropriate limits of integration.

SCRATCH PAPER

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