EGR 265, Math Tools for Engineering Problem Solving
March 4, 2009, 50 minutes

Name: .....

TEST II

Problem 1 (20 points)

Solve the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(0) = -3, \quad y'(0) = 1.$$

Problem 2 (20 points)

Find the general solution of

$$y'' - 2y' + 2y = \sin(2x).$$

Problem 3 (20 points)

Solve the initial value problem

$$y'' + \frac{7}{2}y' - 2y = 18, \quad y(0) = 0, \quad y'(0) = 0.$$

Problem 4 (20 points)

A mass of 10 kilograms stretches an undamped spring by 4.9 meters.
(a) Find the value of the spring constant k.
(b) Find the frequency ω of free oscillations of the spring/mass-system.
(c) Find the equation of motion if the mass is released 1 meter above the equilibrium position at an upward velocity of $\sqrt{2}$ m/s. Assume here that the positive x-direction is oriented downwards.
(d) Write the equation of motion in the form $x(t) = A\sin(\omega t + \phi)$ and determine A and ϕ.
(e) Find the first positive time at which the mass passes through the equilibrium position.

Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is numerically equal to 18 times the instantaneous velocity. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Find the general solution of the 3rd order linear DE

$$y''' - y'' - 2y' = 0.$$