MA 125-8B,   CALCULUS I

April 3, 2008

Name (Print last name first): ....

 Your signature: ....

$\boxed{\boxed{\text{TEST III}}}$

$\boxed{\text{PART I}}$

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Differentiate the function $y = e^{-x} \cos x$.

Answer: ....

Question 2

Find the derivative of the function $f(x) = \dfrac{\ln x}{2x^2}$.

Answer: ....

Question 3

Differentiate the function $y = \ln(\cos x)$.

Answer: ....

Question 4

Suppose $h(x) = e^{-x}g(x)$ where $g(0) = 2$ and $g'(0) = -4$. Find the numerical value of $h'(0)$.

Answer: ....

Question 5

Find the derivative of the function $y = \tan^{-1}(2x)$.

Answer: ....

Question 6

Evaluate $\lim\limits_{x \to \infty} \dfrac{\ln(x + 2)}{3x - 5}$.

Answer: ....

$$\boxed{\textbf{PART II}}$$

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(1) Find the linear approximation of the function $g(x) = \sqrt[3]{1+x}$ at $a = 0$.

(2) Use Newton's method with initial approximation $x_1 = -1$ to find x_2, the second approximation to the root of the equation

$$x^3 + 5 = 0.$$

Problem 2

Consider the function $f(x) = x^3 - 12x$ on the interval $[-3, 3]$.

(1) Find all critical numbers of f in the given interval.

(2) Find the maximum and minimum values of f on $[-3, 3]$.

Problem 3

(1) Differentiate the function, and simplify completely by expressing your answer as a single fraction.

$$g(x) = \ln\left(\sqrt[3]{\frac{x+1}{x-1}}\right).$$

(2) Differentiate the function

$$y = e^{-x\cos x}.$$

Problem 4

(a) Use logarithmic differentiation to find the derivative of the function

$$y = x^{\cos x}.$$

(b) Differentiate the function $y = \sin^{-1}\left(\sqrt{x}\right).$

Problem 5

(1) Find the absolute maximum value of $f(x) = x^3 - 3x$ on the interval $[0, 3]$.

(2) Show that the equation $2x + 8 - \cos x = 0$ has exactly one real root.