Final Exam

Name:_____

Student Number:_____

Show all your work and give reasons for all your answers. Good luck!

Part I

In part I essentially no partial credit is awarded. Hence work these problems carefully and show your work. Each problem in part I is **6** points.

(1) Evaluate $\int \frac{x^3+x^9}{\sqrt{x}}\,dx$.

(2) Evaluate $\int x^2 \sin(x^3+1)\,dx$.

(3) Evaluate $\int xe^x\,dx$.

(4) Evaluate $\int \frac{1}{x^2-1}\,dx$.

(5) Evaluate $\int_0^\infty \frac{1}{x^2+1}\,dx$.

(6) Evaluate $\int \ln(x)\,dx$.

(7) Find the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}.$$

(8) Find the MacLaurin series for the function $f(x) = \frac{1}{x^8+1}$. For which values of x is this series equal to to the function $f(x)$?

(9) Find the angle between the vectors $\mathbf{a} =< 1, -1, 3 >$ and $\mathbf{b} =< -1, 1, 2 >$. (You may leave your answer in the form $\cos^{-1}(\theta)$.)

(10) Find the point of intersection of the line and $\begin{cases} x = -1 + s \\ y = 4 + s \\ z = 1 + 3s \end{cases}$ and the plane $x + y + z = 9$.

(11) **Set up (you do not need to evaluate)** an integral for the arc length of the parametric curve $x(t) = \sin(t^2)$, $y(t) = te^t$, $0 \leq t \leq 5$.

Part II

In part II partial credit is awarded. Hence work these problems carefully. Each problem in part II is **10** points.

(12) Use series to approximate $\int_0^{1/10} \sin(x^4)\,dx$ with an error less than 10^{-15}.

(13) Evaluate $\int \sin(\sqrt{x})\,dx$.

(14) **Set up (but do not evaluate)** an integral for the volume of revolution of the area which is bounded by the lines $x = 1$, $x = 3$ and the graphs of the functions $f(x) = x$ and $g(x) = x^2 + 4$, and is rotated about the line $x = -5$.

(15) Find the line of intersection of the planes $x + 2y - z = 1$ and $-2x + y + z = 2$.