Calculus II Final Exam. Spring 2005. NAME:_____

Write clearly and organize your work. Justify conclusions. No books, notes, or calculators allowed.

PART I: Short problems. Simplify and circle answers. No partial credit. Five points each.

1. Put in correct partial fraction format, but do not solve for the unknown constants $\frac{x+1}{x^2(x^2+4)(x+3)}$.

2. Let $g(x) = \int_0^{\cos(x)} \frac{1}{\sqrt{s^2+1}}\, ds$ and find the derivative $g'(x)$.

3. Evaluate $\int \cos^2(cx)dx$, $c \neq 0$.

4. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{5^n} = 1 + \frac{1}{5} + \frac{1}{25} + ...$

5. Find symmetric equations for the line containing the point $(1, 2, 3)$ and perpendicular to the plane $2x + 3y + 5z = 0$.

6. Evaluate $\int \frac{\sin x}{5 - \cos x}\, dx$

7. Find the area of the parallelogram formed by the vectors $\overrightarrow{A} =< 1, 1, 1 >$ and $\overrightarrow{B} =< 2, 3, 2 >$.

8. Let $\overrightarrow{V} =< 4, -2, 1 >$ and $\overrightarrow{W} =< 1, 2, 3 >$. Find the vector projection of $\overrightarrow{V}$ onto $\overrightarrow{W}$.

PART II. Longer problems. 10 points each. Justify conclusions with appropriate math steps or arguments. Simplify and circle answers. Partial credit possible.

1. Evaluate $\int_0^7 x\sqrt{x+9}dx$

2. Evaluate $\int xe^{4x}dx$

3. Evaluate $\int \frac{1}{x^2+5x+6}\,dx$

4. Evaluate $\int_0^\infty \frac{e^{-x}}{1+e^{-x}}\,dx$

5. Find the volume of the solid obtained by rotating about the $y-$axis the region bounded by the curves $y = x^3, x = 0$, and $y = 27$. Also sketch the region.

6. Let $g(x) = \frac{1}{2}\ln(\frac{1+x}{1-x})$ for $|x| < 1$. Then $g'(x) = \frac{1}{1-x^2}$; use this and the geometric series to find the Maclaurin's series of $g(x)$.

7 Let $f(x) = \frac{1}{x}$.

(A.) Find the second degree Taylor polynomial $T_2(x)$ for $f(x)$at $a = 2$.

(B) Use Taylor's inequality (the idea often used in class) to estimate the maximum error of $|R_2(x)| = |f(x) - T_2(x)|$ for $|x - 2| \leq 1$.

8. Find the interval of convergence for the series:

(A) $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2 2^n}$

(B) $\sum_{n=0}^{\infty} (-1)^n \frac{5^n x^n}{n!}$

9. Let $P : (2, 1, -3)$,$Q : (1, -1, 1)$,$R : (1, 2, 2)$ be three points.
(A) Find a vector $\overrightarrow{N}$ normal to the plane containing P, Q,and R.

(B) Find an equation of the plane containing P, Q,and R.

10. Find the line of intersection for the planes $3x+2y+3z = 0$ and $x+3y-2z = 0$.

11. Find an equation of the plane that contains the parallel lines

$$\frac{x}{2} = \frac{y}{4} = \frac{z}{3} \text{ and } \frac{x+1}{2} = \frac{y-1}{4} = \frac{z-1}{3}.$$

12. Find the line through the point $(1, 1, 1)$ which is perpendicular to the line $\overrightarrow{r}(t) = t < 2, 1, -2 >$.

Extra Credit

1.) (10pt) Find a positive number c such that $\sum_{n=0}^{\infty}(2+c)^{-n} = c$.

2.) (10pt) Find the Maclaurin's series of $g(x) = \frac{\ln(x+1)}{x}$.