Calculus I

TEST 2A

March 3rd, 2005

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. *(10pts)* Find $f''(x)$

$$f(x) = (5 - x^2)^4$$

2. *(10pts)* Differentiate

$$y = \ln \sqrt[3]{\frac{\cos x}{2x + 1}}$$

3. *(10pts)* Differentiate

$$y = xe^{\sin x}$$

4. *(10pts)* Differentiate

$$y = x^{\tan x}$$

(Hint: use logarithmic differentiation)

5. *(10pts)* Find the equation of the tangent line at the point $(1, -\frac{1}{2})$ for the following curve:

$$y = f(x) = \frac{2 - 3x}{1 + x^2}$$

6. *(8pts)* Suppose that $h(x) = f(g(x))$. Find $h'(1)$ if $f'(5) = 2$ and $g(x) = 5e^{x-1}$.

7. *(10pts)* Differentiate

$$y = \sin\left(\arctan\left(x^2\right)\right)$$

8. (a) *(7pts)* Find $\frac{dy}{dx}$ by implicit differentiation.

$$y^2 + 2xy + 3x^2 = 24$$

(b) *(5pts)* Find the points on the curve where the tangent line is horizontal.

9. (a) *(6pts)* Let $f(x) = x^3 - 3x + 4$. Starting with $x_1 = -2.2$, use Newton's method to compute x_3.

(b) *(2pts)* Starting with $x_1 = -2.2$, at what x_n can you stop, if you would like to know the solution of $x^3 - 3x + 4 = 0$ correct to 4 decimal places? Explain your answer.

(c) *(2pts)* Give an example of a choice of x_1 for which Newton's fails. Explain why Newton's method fails for that choice.

10. (a) *(6pts)* Let $f(x) = \sqrt[3]{3x^2 + 5}$. Find the linear approximation of $f(x)$ at $x = 1$.

(b) *(4pts)* Use the above linear approximation to estimate $f(0.99)$.