Calculus I

TEST 1A

February 3rd, 2005

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. (15pts) Evaluate the following limits.

(a) (5pts) $\lim_{x\to -3} \frac{x^2+5x+6}{x+3}$

(b) (5pts) $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

(c) (5pts) $\lim_{x\to \infty} \frac{2x^2+9-3x}{2+x^2}$

2. (10pts) Evaluate the following limits. If limit is infinite or does not exist, say so.

(a) (5pts) $\lim_{x\to -2^+} \frac{3+x^2}{x+2}$

(c) (5pts) $\lim_{x\to \infty} \sqrt{x^2+3x} - \sqrt{x^2-x}$

3. (10pts) Sketch the graph of a function f that satisfies all of the following conditions:
$f'(5) = -1$, $\lim_{x\to\infty} f(x) = 5$, $\lim_{x\to -\infty} f(x) = -1$,
$\lim_{x\to 1^+} f(x) = -3$, $\lim_{x\to 1^-} f(x) = 2$, $f(1) = -1$.

4. (10pts) Let the function $f(x)$ be given by

$$f(x) = \begin{cases} -cx + 13 & \text{if } x \leq 2 \\ (x-c)^2 & \text{if } x > 2 \end{cases}$$

Find the value(s) for c that make f continuous at $x = 2$.

5. (15pts) Consider the function $f(x) = \frac{-x}{x+2}$.
(a) (8pts) Find $f'(x)$ by using the definition of the derivative.

(b) (7pts) Find an equation for the tangent line to the graph of $y = f(x) = \frac{-x}{x+2}$ at the point (-3,-3). (If you are unable to obtain the answer to part (a) you may use that $f'(-3) = -2$).

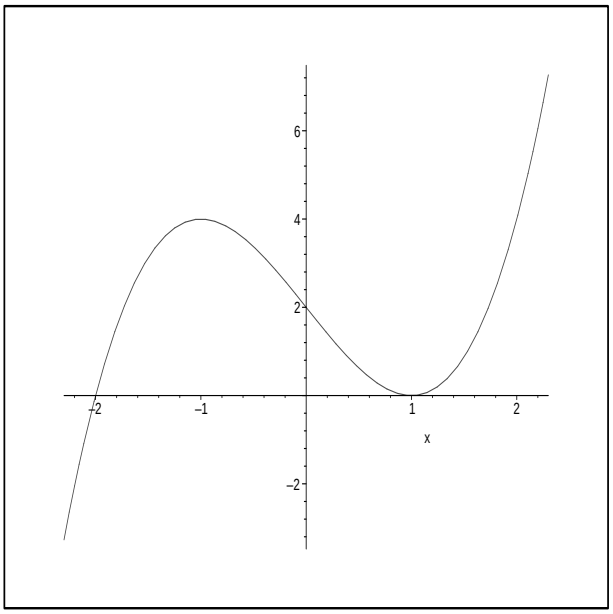
6. (10pts) Prove that the equation $x^5 - 3x + 10 = 0$ has a solution. State the name of the Theorem(s) you are using.

7. (12pts) A person stands on top of a building and throws a ball upward vertically. The height of the ball in meters above the ground after t seconds is given by $s(t) = 40 + 10t - 5t^2$.
(a) (6pts) Using the definition of the derivative, find the velocity of the ball at time t.

(b) (3pts) At what time does the ball have a velocity of 5 m/s?

(c) (3pts) When does the ball hit the ground? What is the velocity when the ball hits the ground?

8. (18pts) The graph of $y = f'(x)$ is given below. Note that this is **not** the graph of $y = f(x)$.



(a) (3pts) On what intervals is f decreasing or increasing?

(b) (3pts) At what values of x does f have a local maximum or minimum?

(c) (3pts) Where is the graph of f concave upward or downward?

(d) (3pts) Sketch a graph of $f''(x)$.

(e) (3pts) State the $x-$coordinate(s) of the point(s) of inflection of f.

(f) (3pts) If $f(0) = 0$, sketch a possible graph of f