Mathematics 125 **Midterm 1**
Feb. 5, 2004

- Calculators are allowed *only* for numerical calculations, that is you may not graph functions on your calculator.

- There are two sheets of scratch paper attached at the end of the exam. Use them and but do not tear them off the exam.

- Show your work; clearly write down each step in your calculations/reasonings. *No credit* is given to a correct numerical answer *without* any justification.

1.(25 pts) Find the following limits *if* they exists. The values of the limits may take $+\infty$ or $-\infty$. Justify your answers for each case.

a)

$$\lim_{x \to 1} \frac{1}{x^2 - 4}.$$

b)

$$\lim_{x \to 2^+} \frac{1}{x^2 - 4}.$$

c)

$$\lim_{x \to 2} \frac{x-2}{x^2-4}.$$

d)

$$\lim_{x \to \infty} \sqrt{x+1} - \sqrt{x}.$$

e)

$$\lim_{x \to +\infty} \frac{x^2 + 4}{x^2 - 4}$$

2.(20 pts) **a)** Show that the function $|x-6|$ is not differentiable at $x=6$.

b) Find a formula for $f'(x)$ where it is defined, and sketch its graph.

3.(20 pts) **a)** f is defined as follows;

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show f is continuous at $x = 0$. (Hint: use the Squeeze Theorem)

b) g is defined as follows;

$$g(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Show g is differentiable at $x = 0$. (Hint: use **a)**)

4.(15 pts) Show that there exists a number x whose cube is exactly one more than its square.

5.(20 pts) **a)** Use the definition of a derivative to find $f'(4)$, where $f(x) = \sqrt{x}$.

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b) Find an equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(4, 2)$.

6.(20 pts) Water is flowing into three containers of different shapes; A) pinched neck, B) cylindical and C) conical (point-down). Let $H_A(t)$ represent the height of the water level of the container A at time t, and $H_B(t)$ $H_C(t)$ defined accordingly.

a) For each function, select a graph which best represents its behavior from the six graphs below.

b) For the derivative of each function (that is, $H'_A(t)$, $H'_B(t)$ and $H'_C(t)$) select a graph which best represent its behavior from the six graphs below.

Remark: You may want to compare the answers from a) and b) to see they are indeed consistent.