Calculus III Test 2 Feb. 25, 2003 NAME_____________________________________

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. All problems will be of equal value. Be sure to work the given problem; otherwise you will not receive credit.

1. Find an equation for the plane tangent to $z = xy$ at the point $(x, y, z) = (2, 3, 6)$.

2. Find an equation for the plane tangent to the surface defined by $x^2 + 3y^2 + 2z^2 = 6x + 4y + z$ at the point $(x, y, z) = (3, 1, -2)$.

3. Let g be a twice differentiable function of a single variable. Find all values of c so that $u(x, t) = g(x + ct)$ is a solution of the wave equation $u_{tt} = 3u_{xx}$

4. Suppose $x^2 + y^2 + z^2 - xyz = 1$. Find $\partial z/\partial x = z_x$.

5. Let $f(x, y) = \ln(x^2 + 3y^2)$. Find the directional derivative of f at the point $(1, 2)$ in the direction toward the origin.

6. Let $w = \frac{y}{x} + \frac{x}{z}$. Find the maximum rate of change of w at $(x, y, z) = (2, 1, -1)$ and the direction in which it occurs.

7. Let $f(x, y) = 3x^2 + 2xy + y^2 - 3x + y$. Find all critical points and use the second derivative test to identify local minima, maxima, and saddle points.

8. Let $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$. Find all critical points and use the second derivative test to identify all local minima, maxima, and saddle points.

9. Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = 2x + 3y - 4z$ on the sphere $x^2 + y^2 + z^2 = 1$.

10. Use Lagrange multipliers to find the area of the largest rectangle which can be inscribed in the general ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

EXTRA CREDIT: :

(A) Let $g(x, y) = (y^2 - 1)^2 + (y^2x - y - 1)^2$. Find all critical points, and identify each as a point of local minimum, maximum, or saddle.