MA 126, Final Exam

May 16, 2003

1. (4 each) Find the following: (show all work)

a) $\int x^3 \ln x\, dx$

b) $\int_0^2 x^2 \sin\left(x^3\right) dx$

c) $\int \frac{1}{x^2-4} dx$

d) $\int x \cos x\, dx$

e) $\int_0^1 \left(x^2 + 1\right)^2 dx$

f) $\int \frac{x^2}{1+x^6} dx$

2. (6) Find an infinite series which represents the following: (show all details)

$$\int_0^2 e^{x^3} dx$$

3. (6) Define the definite integral from a to b of f.

4. (8) Does $\int_1^\infty x^{5/6} e^{-x^2} dx$ converge? Show why or why not.

5. (8) Find the first 6 terms of the series solution to the equation

$$
\begin{aligned}
Y'' + 2Y &= \sin x \\
Y(0) &= 2, \\
Y'(0) &= 1
\end{aligned}
$$

6. (8) Estimate

$$\int_0^{\frac{1}{2}} e^{x^2}\,dx$$

to within an accuracy of .01. Show your estimate is within this accuracy.

7. (8) Does the series

$$\sum_{i=1}^{\infty} \frac{(-1)^i \, 4^i}{i!}$$

converge? Why or why not, in detail?

8. (8) Rotate the region between $y = 2x^2 - x$ and $y = x^3$ about the y axis. What is the volume?

9. (8) Estimate $\sin\left(\frac{6}{5}\right)$ to within an accuracy of .001, using Taylor series about zero. Show your estimate is within this accuracy. You can leave your estimate as a sum.

10. (8) Does the following series converge or diverge? $\sum_{i=1}^{\infty} \frac{i^{\frac{3}{2}}}{i^3+1}$? You must prove all statements about specific series (such as $p-$series) which you make in this problem.

11. (8) A dam is in the shape of the curve: $y = x^4$. x goes from 0 to 5. At the bottom of the dam, a region in the shape of the area inside the parabola $y = 2x^2$, for x between 0 and 1, was inadvertantly built with defective cement. We need to know whether the dam will burst, so we need to the force due to hydrostatic pressure on this region as a function of the depth h of the water. There is always at least two units of water in the dam, but h can fluctuate from two units up to half the height of the dam, depending on the amount of water in the lake behind it. We have chosen the units of length so that the density of water is 1. It is known that the dam bursts when the force due to hydrostatic pressure reaches 100 units of force. Does it burst? If so, what is the depth when it bursts? Show all work.