MA-125 Calculus - I. THE FINAL EXAM

I. (9%) The graph of a function $f(x)$ is shown below:

a) Find the interval where $f'(x)$ is positive.
b) Find the interval where $f'(x)$ is negative.
c) Where the $f'(x)$ is zero?
d) Sketch the graph of the derivative on the same figure. Try you best to make it up to scale.

II. (9%) Find the derivative of the function

$$y(x) = (x^2 + 1)e^x.$$

III. (9%) Find the derivative of the function

$$y(x) = \frac{\sin x}{1 + x^3}.$$

IV. (9%) Find the derivative of the function

$$y(x) = e^{-x} + \tan x.$$

V. (9%) Find the derivative of the function

$$y(x) = (1 + 2x)^8 + \cos(5x).$$

VI. (9%) Let

$$y(x) = x^4 - 4x + 2$$

a) Find the equation of the tangent line to the curve at the point $x = 0$.
b) Find the point(s) where the tangent line is horizontal.

VII. (9%) Let

$$y(x) = x^4 + x.$$

a) Find the differential dy.
b) Evaluate dy and Δy given that $x = 2$ and $dx = \Delta x = 0.1$.
c) Sketch a diagram showing the line segments with lengths dx, dy, and Δy

VIII. (10%)

a) Formulate the Mean Value Theorem.

b) Give the Geometric interpretation of the theorem.

c) Give the Mechanical interpretation of the theorem.

d) Explain why it is necessary to state in the formulation of the theorem that the function is differentiable.

IX. (9%) The function $f(x)$ is given by the formula:

$$f(x) = 3x^4 - 4x^3 + 1.$$

1) Find $f'(x)$.
2) Find the critical point(s).
3) Find the intervals on which $f(x)$ is increasing and decreasing.

X. (9%) The function $f(x)$ is given in the previous problem: $f(x) = 3x^4 - 4x^3 + 1$.
4) Find $f''(x)$.
5) Determine the concavity of the curve.
6) Find the inflection point(s).
7) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
8) Show the curve's general shape, basing on the results obtained in 1) – 7).

XI (9%) Evaluate the definite integral:

$$\int_1^4 \frac{x^3 - 1}{\sqrt{x}} dx.$$