MA 125-8C, Spring 2003

# TEST # 3

## April 22, 2003 (105 minutes)

Name: SSN:

Max. Points: 100 + 10 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they demonstrate which method you used to get the result. You may use separate sheets for this.

The test is **closed book** and **closed notes**. You may use a calculator.

**1.** Determine the following integrals ($5 + 5 + 5$ points):

(a) $\displaystyle\int_{-2}^{-1} \frac{1}{x}\, dx$

(b) $\displaystyle\int_{1}^{4} \frac{t^2 - t}{\sqrt{t}}\, dt$

(c) $\displaystyle\int_0^1 \frac{3}{x^2+1}\,dx$

**2.** Calculate the following limits $(5+5+5$ points):

(a) $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

(b) $\displaystyle\lim_{x\to 0^+} \sqrt{x}\ln x$

(c) $\displaystyle\lim_{x\to \pi/2} \frac{1-\cos x}{\sin x}$

**3.** Consider the function $f(x) = x \sin x + \cos x$.

(a) Find the absolute maximum and absolute minimum of $f(x)$ in the domain $-\frac{\pi}{2} \leq x \leq \pi$. (10 points)

(b) Show that there exists a number $c$ between 0 and $\pi$ such that $f'(c) = -\frac{2}{\pi}$. (5 points)

**4.** For the function $f$ with the graph sketched below find $\int_0^3 f(x)\,dx$, $\int_3^5 f(x)\,dx$ and $\int_0^5 f(x)\,dx$. (6 points)

**5.** For the function $f(x) = 3x^4 - 16x^3 + 24x^2 - 32$ find the intervals of increase or decrease as well as the intervals where the function is concave upward or concave downward. Also find all local maxima, local minima and inflection points. Use this information to sketch the graph of $f$. (15 points)

**6.** Consider the function $f(x) = \sqrt{x^2 - 1}$ on the domain $1 \leq x \leq 4$.

(a) Find the Riemann sum $R_3$ for $f$ with $n = 3$, taking sample points to be right endpoints. (5 points)

(b) Is $R_3$ an overestimate or an underestimate for $\int_1^4 \sqrt{x^2 - 1}\, dx$? Why? (4 points)

**7.** An open box with a square base is constructed and must have a surface area of 1200 square inches. What are the dimensions (height and base length) of the box which guarantee maximal volume? (10 points)

**8.** A particle moves along a straight line with constant acceleration $a(t) = 2$ m/sec$^2$. Its position and velocity at $t = 0$ are $s(0) = 2$ m and $v(0) = -2$ m/sec.

(a) Find the particle's position at time $t = 10$ seconds. (8 points)

(b) Find the total distance traveled by the particle between $t = 0$ and $t = 10$ seconds. (7 points)

**9.*** Find the point on the parabola $y = 1 - x^2$ at which the tangent line cuts from the first quadrant the triangle with the smallest area. (10 points bonus)