MA 125-8C, Spring 2003

TEST # 1

February 6, 2003 (105 minutes)

Name: SSN:

Max. Points: 100 + 10 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations** and mention the **rules and theorems** which were used. You may use separate sheets for this, if necessary.

To receive credit, all solutions have to be based on the **methods from Chapter 2** of Stewart's book.

The test is **closed book** and **closed notes**. You may use a calculator.

1. Evaluate the limits: (5P+5P+5P*)

(a) $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

(b) $\lim_{x\to \infty} \dfrac{2x^2-1}{x^2-x}$

(c)* $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{x+x^2}\right)$

2. Provide a possible graph of a function $y = f(x)$ with the following properties (15P):

(i) $\lim_{x \to 1^+} f(x) = \infty, \quad \lim_{x \to 1^-} f(x) = -\infty$

(ii) $\lim_{x \to \infty} f(x) = 2, \quad \lim_{x \to -\infty} f(x) = 0$

(iii) f is continuous at 2, but not differentiable at 2.

3. Show that the equation $x^5 + x^2 = 1$ has a solution x between 0 and 1. Justify your answer by quoting the theorem which was used. (10P)

4. Consider the function $f(x) = \frac{1}{x-1}$.

(a) Find $f'(x)$ by using the definition of derivative. (10P)

(b) Find an equation for the tangent to the graph of $y = \frac{1}{x-1}$ at the point $(0, -1)$. (10P)

5. If a ball is thrown up vertically with an initial velocity of 10 meters per second, then after t seconds its height in meters above ground is given by the formula $s(t) = 10t - 4.9t^2$.

(a) Find the velocity and the acceleration of the ball at time t. (10P)

(b) At what time does the ball have velocity 8 m/s? (5P)

(c) At what time does the ball reach its maximal height, before starting to fall back to the ground? What is the maximal height? (5P*).

6. For a function f the graph of its derivative f' is given below:

(a) Sketch the graph of the second derivative f'' (10P).

(b) Find all the intervals in which the function f is increasing, decreasing, concave upward, or concave downward. Also find the values of x where f either has local maxima or minima, or inflection points (10P).

(c) Sketch a possible graph of f given that $f(0) = 0$. The graph should reflect all the properties found in (b). (10P)