Instructor:_____ Name:_____

# Exam IV

**Calculus I; Fall 2011**

# Part I

**Part I consists of 10 questions, each worth 6 points. Clearly show your work for each of the problems listed.**

(1) Let $f(x) = 3x^4 + 4x^3 - 12x^2$. Find all local/absolute max/min of $f(x)$. State both $x$ and $y$ coordinates.

(2) Find the absolute max/min of $f(x) = x^5 - 1$ on the interval $[-1, 1]$. Give both $x$ and $y$-coordinates and justify your answer.

(3) Find two positive numbers whose sum is 10 and whose product is maximal. (**You must justify your answer!**)

(4) Find the number $c$ whose existence is guaranteed by the Mean Value Theorem for the function $y = f(x) = x^2$ on the interval $[-1, 2]$.

(5) If $f'(x) = (x-3)^4(x+5)^5$. **Note that you are already given the derivative $\mathbf{f'(x)}$.** Find all critical points, where $f(x)$ is increasing and decreasing, and also find the $x$-coordinate(s) of all local max/min.

(6) Find the most general **anti-**derivative of $f(x) = \frac{x^2 \sin(x)+x^3}{x^2}$.

(7) Find the most general **anti-**derivative of $f(x) = \sqrt{x}(x+1)$.

(8) Find all asymptotes of the function $\frac{1-x^2}{(x+2)(x-3)}$.

(9) If the acceleration is given by $a(t) = t^2$, $v(0) = 1$ and $s(0) = 2$, find $S(2)$.

(10) Find the most general **anti-** derivative of $y = f(x) = \frac{1}{\sqrt{1-x^2}} + \frac{1}{x}$

# Part II

**Part II consists of 3 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.**

(1) **[10 pts]** Find the absolute max/min of the function $f(x) = (x^2 - 2x)^3$ on the interval $[-2, 2]$.

(2) Given the function $f(x) = \frac{(x^2-4)}{(x+1)^2}$

(a) **[2 pts]** Find the $x$ and $y$ intercepts of the function.

(b) **[3 pts]** Find all asymptotes.

(c) **[4 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(d) **[2 pts]** Find the local maximum and local minimum value(s) of $f(x)$. (Be sure to give the $x$ and $y$ coordinate of each of them).

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) **[2 pts]** Find all points of inflection (be sure to give the $x$ and $y$ coordinate of each point).

(g) **[5 pts]** Use the above information to graph the function **on the next page**. Indicate all relevant information in the graph.

Put the graph of Problem 2 on this page.

(3) **[10 pts]** An oil rig is located 2 $km$ off shore at point A. The closest point $B$ on the shore is 15 $km$ from an oil refinery (which is also located on the shore). If it costs \$100/km to lay a pipe line in the ocean and \$5/km to lay a pipe line on land, determine the cheapest way to lay a pipe line from the oil rig to the refinery.