MA 125 6C, CALCULUS I

September 25, 2017

Name (Print last name first): .....

TEST I

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = \frac{1}{x}$ is $f'(x) = \frac{-1}{x^2}$.

Question 2

Find the derivative of $f(x) = x^2(x^2 + \sqrt[3]{x})$

Answer: .....

Question 3

Find the derivative of $y = f(x) = x\cos(x)$.

Answer: .....

Question 4

Find the derivative of $y = f(x) = \dfrac{x}{x^2+1}$.

Answer: .....

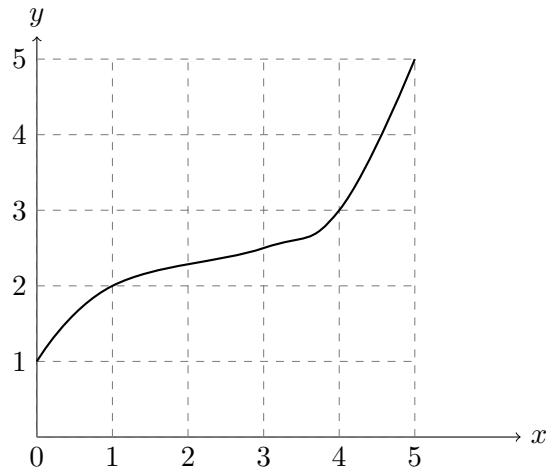
Question 5

Find the equation of the tangent line to the graph of $y = f(x) = 3x^2 + x$ at the point $x = 1$.

Answer: .....

Question 6

Using the graph of the function $y = f(x)$ below estimate: $f(2)$ and the derivative $f'(2)$.



Answer: .....

Question 7

If $Q(x) = x^3 - 25x + 100$ is the cost per item of producing x-items. Is the cost per item increasing or decreasing when you produce 15 items?

Question 8

Show that the equation $f(x) = 2x^3 + 3x + 6 = 0$ has at least one solution; find an interval of length $\frac{1}{2}$ which must contain a solution.

PART II

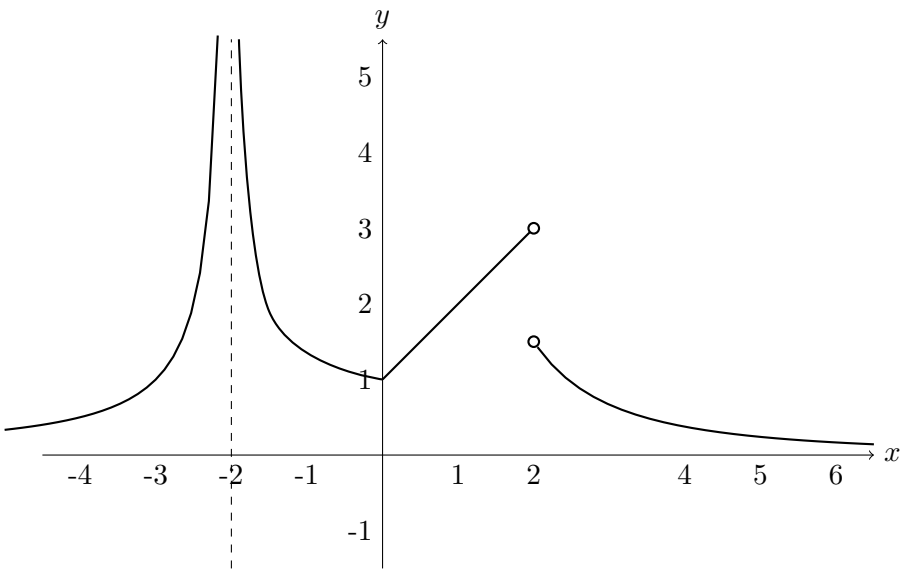
Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (12 points)

Suppose that $S(t) = t^3 - 12t$ $m.$ is the position of a particle at time t (in seconds) on a line.
(a) Is the position increasing or decreasing at time $t = 1$?
(b) What is the displacement from $t = 0$ to $t = 3$.
(c)What is the total distance travelled between $t = 0$ and $t = 3$.

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:



1. $\lim_{x \to -2^-} f(x) =$
2. $\lim_{x \to -2^+} f(x) =$
3. $\lim_{x \to -2} f(x) =$
4. $\lim_{x \to 2^-} f(x) =$
5. $\lim_{x \to 2^+} f(x) =$
6. $\lim_{x \to 2} f(x) =$
7. $\lim_{x \to \infty} f(x) =$
8. $\lim_{x \to -\infty} f(x) =$
9. State all intervals on which $f(x)$ is continuous.
10. State all intervals where $f(x)$ is differentiable.

Problem 3 (12 points)

Evaluate the following limits:

a) $\lim_{x\to\infty} \sqrt{x^2+1} + \sqrt{x^2+2}$

b) $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2+2}$

c) $\lim_{x\to\infty} x\sin(x)$

Problem 4 (10 points)

Define a function

$$f(x) = \begin{cases} \frac{x^2+x-2}{x-1}, & x \neq 1, \\ k, & x = 1, \end{cases}$$

a) (8 points) Find the value of k so that $f(x)$ is a continuous function for all real numbers x.

b) (2 points) For the value of k you found, is $f(x)$ differentiable at $x = 1$? Briefly explain.