MA 126 - 8C CALCULUS II
September 16, 2014

Name (Print last name first): .....

Student Signature: .....

TEST I

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. Limited partial credit is awarded for this part of the test!

Each question is worth 8 points.

Question 1

Find an equation of the sphere with center $(1, 0, 1)$ that passes through the point $(1, 1, 1)$.

Answer: .....

Question 2

Find the angle between the vectors $\mathbf{u} =< 0, 1, 0 >$ and $\mathbf{v} =< \sqrt{3}, 1, 0 >$. (Write your angle in degrees or radians or leave it in terms of $\cos^{-1} = \arccos$!)

Answer: .....

Question 3

Find the parametric equations of the line that is perpendicular to the plane $2x - 3y - 7z = 1$ and passes through the point $(1, 1, -2)$.

Answer: .....

Question 4

Find an equation of the plane that contains both the point $P(1, 1, 1)$ and the vectors $\mathbf{u} = \langle 0, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, 1, 0 \rangle$.

Answer: .....

Question 5

Find the distance from the point $P(1, 2, 1)$ to the plane $x + 2y + 2z = 1$.

Answer: .....

Question 6

Find the point of intersection of the plane $x + y - z = -1$ and the line given by the parametric equations

$$\ell_1 := \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + t \end{cases}$$

Answer: .....

PART II

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

This problem has two separate questions (a) and (b). Answer each question.

(a) A constant force with vector representation $\mathbf{F} = 6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ moves an object along a straight line from the point $P(1, 1, 2)$ to the point $Q(3, 1, 3)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

(b) A (tiny horizontal) bicycle pedal is pushed by a foot with an 80 pound-force making an angle of 30° with the horizontal line/plane. The shaft of the pedal is one half (i.e., 1/2) foot long and is in a horizontal position. Find the magnitude of the torque.

Problem 2

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(1,0,0)$, $Q(0,-2,0)$ and $R(0,0,3)$.

(b) Find the volume of the box generated by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Problem 3

Consider the two lines given by the parametric equations

$$\ell_1 = \begin{cases} x = 2t \\ y = -2 + 2t \\ z = 4 - t \end{cases} \quad \text{and } \ell_2 = \begin{cases} x = 10s \\ y = 3 + 5s \\ z = -3 + 2s \end{cases}$$

(a) Determine whether they are parallel.

(b) Determine whether they intersect. If they do intersect, find the point of intersection.

(c) Determine whether they are skew.

Problem 4

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the parametric equations of the line of intersection of the planes

$$x + y + z = 3 \quad \text{and} \quad x - 2y + 3z = 0.$$

(b) Find the distance from the point $P(3, 2, 1)$ to the plane $z = 0$.

SCRATCH PAPER

(Scratch paper will not be graded!)

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