MA 126 - 8C   CALCULUS II
November 20, 2014

Name (Print last name first): ....

Student Signature: ....

$\boxed{\text{TEST IV}}$

$\boxed{\text{Closed book - Calculators and One Index Card are allowed!}}$

$\boxed{\text{PART I}}$

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. <u>Limited partial credit is awarded for this part of the test</u>!

Each question is worth 8 points.

<u>Question 1</u>

Determine whether the **sequence** $a_n = \dfrac{n+1}{n+2}$ converges or diverges. If it converges, find its limit.

Answer: ....

<u>Question 2</u>

Determine whether the infinite **series** $\displaystyle\sum_{n=1}^{\infty} \dfrac{5}{n(n+1)}$ is convergent or divergent.

Answer: ....

Question 3

Determine whether the **geometric series** $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^{n-1}$ is convergent or divergent. If it is convergent, find its sum.

Answer: ....

Question 4

Determine whether the infinite **series** $\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n+3}$ is convergent or divergent.

Answer: ....

Question 5

Use the integral test to determine whether the infinite **series** $\sum_{n=2}^{\infty} \frac{1}{n \, (\ln(n))^2}$ is convergent or divergent. [You are not required to show your work here!]

Answer: ....

Question 6

Determine whether the **alternating series** $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+2}{n^3+2}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific here!)

Answer: ....

$\boxed{\textbf{PART II}}$

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Determine whether or not each of the following **sequences** converges as $n \to \infty$. (Give reasons!)

$$a_n = (-1)^n \frac{1}{n+1}$$

$$b_n = \cos(n\pi)$$

$$c_n = \frac{\ln(n)}{n}$$

Problem 2

Find the <u>radius</u> and <u>interval</u> of convergence of the **power series**

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x - 5)^n.$$

Be sure to check any endpoints that exist!

Problem 3

Answer all the following questions.

(a) Find a power series representation of the function

$$f(x) = \frac{1}{1 + x^2},$$

and state the interval on which your series converges to the function.

(b) Use the series in (a) to find a power series representation of the function

$$g(x) = \frac{x^2}{1 + x^2}.$$

Problem 4

Answer all the following questions.

(a) Find the Maclaurin series representation of the function $f(x) = e^{-x}$.

(Hint: The Maclaurin series of e^x might prove useful here, if need be!)

(b) Use the series in (a) to write out the Maclaurin series expansion of $e^{-0.1}$.

(Do <u>not</u> compute and add the terms of your series!)

(d) Find the <u>minimum</u> number of terms you need in the series in (b) to approximate $e^{-0.1}$ with an error less than $10^{-3} = \dfrac{1}{1000} = 0.001$? (Show your work!)

SCRATCH PAPER

(Scratch paper will not be graded!)

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