MA 126 CALCULUS II
Wednesday, December 10, 2014

Name (Print last name first): .....

Student Signature: .....

Section: ..... Instructor Name: .....

FINAL EXAM

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 10 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. Limited partial credit is awarded for this part of the test. CHECK YOUR ANSWERS!

Each question is worth 4 points.

Question 1

Find the angle between the vectors $\mathbf{u} =< 1, -1, 1 >$ and $\mathbf{v} =< -1, 1, -1 >$. (Give the angle in degrees or radians.)

Answer: .....

Question 2

Find the area of the parallelogram generated by the vectors $\mathbf{u} =< 1, 0, 1 >$ and $\mathbf{v} =< 0, 1, 1 >$.

Answer: .....

Question 3

Find the parametric equations of the line containing the points $P(1, -2, 1)$ and $Q(2, 0, 4)$.

Answer: .....

Question 4

Find an equation of the plane that passes through the point $P(1, -1, 0)$ and is perpendicular (normal) to the line with symmetric equations $\dfrac{x-2}{3} = \dfrac{y+5}{2} = \dfrac{z-1}{-1}$.

Answer: .....

Question 5

Use the Fundamental Theorem of Calculus to find the derivative of the function $g(x) = \displaystyle\int_0^x \sin(t^3)\, dt$.

Answer: .....

Question 6

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{\sqrt{x^3}}\,dx$$

Answer: .....

Question 7

Find the area of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$.

Answer: .....

Question 8

Evaluate the indefinite integral $\int \sin^3(x)\cos(x)\,dx$.

Answer: .....

Question 9

Evaluate the indefinite integral $\int \frac{x}{x+2}\, dx$.

Answer: .....

Question 10

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n^2+2}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific!)

Answer: .....

PART II

Each problem is worth 12 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers. CIRCLE YOUR ANSWER!

Problem 1

Two planes are given by the equations $x + y + z = 1$ for the plane $\mathcal{P}_1$ and $x - y + z = 1$ for the plane $\mathcal{P}_2$.

(a) Find the coordinates of a point of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

(b) Find the normal vector (i.e., the vector perpendicular) to the plane $\mathcal{P}_1$ and the normal vector to the plane $\mathcal{P}_2$.

(c) Find the parametric equations of the line of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

Problem 2

This problem has two separate questions. (Answer all the questions!)

(a) Find the length of the arc of the circular helix with vector equation $\mathbf{r}(t) = \langle \cos(t), \sin(t), 3t \rangle$ when $0 \leq t \leq 3$.

(b) Determine whether the (improper) integral

$$\int_0^\infty 2x\, e^{-x^2}\, dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

Problem 3

Evaluate the following integrals (clearly show the techniques of integration you use):

(a) $\int \frac{1}{x\ln^3(x)}\,dx$

(b) $\int x\,e^x\,dx$

(c) $\int \frac{3x^2+5x+3}{(x+2)(x^2+1)}\,dx.$

Problem 4

This problem has two separate questions. (Answer all the questions!)

(a) Find the **area** of the region enclosed by the parabola $y = x^2 - x$ and the parabola $y = x - x^2$.

(b) The region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the x-axis. Find the **volume** of the solid obtained in this way.

Problem 5

This problem has two separate questions. (Answer all the questions!)

(a) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} (x-3)^n.$$

Be sure to check any endpoints that exist!

(b) Use the Maclaurin series of the function $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ to first write out the Maclaurin series for the function

$$g(x) = \sin(x^2),$$

and then write out the Maclaurin series expansion of

$$\int_0^1 \sin(x^2)\, dx.$$

(Do not compute and add up the terms of your series!)

Using the above information, find the minimum number of terms needed to approximate $\int_0^1 \sin(x^2)\, dx$ with an error less than $0.0001 = \frac{1}{10,000}$.

DO NOT ENTER ANY PROBLEM SOLUTIONS OR WORK ON THIS PAGE.

Summary of scores on problems - for grading purposes only.

	Points
Part I	
Questions 1 – 10	
Part II	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total Exam Score	

SCRATCH PAPER

(Scratch paper will not be graded)

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