EGR/MA265, Math Tools for Engineering Problem Solving
October 28, 2013

Name and section: _____

Instructors name: _____

1. **Do not open this exam until you are told to do so.**

2. This exam has 6 pages including this cover. There are 6 questions, for a total of 100 points. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. **Organize your work,** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.

6. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.

8. **Turn off all cell phones**, and remove all headphones.

Do not write in the table below.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	10	
6	10	
Total:	100	

1. [20 points] Solve the initial value problem

$$y'' - 6y' + 10y = 0, \quad y(0) = -1, \quad y'(0) = 1.$$

2. [20 points] Find the general solution of

$$y'' - 2y' + y = x - 1.$$

3. [20 points] Find the general solution of

$$y'' - 5y' = 2e^{-3x}.$$

4. [20 points] A mass of 10 kilograms stretches an undamped spring by 1 meter and 9 centimeters.

   (a) Find the value of the spring constant k assuming the **gravity constant** $g = \mathbf{9.81m/sec^2}$.

   (b) Find the angular frequency ω of free oscillations of the spring/mass-system.

   (c) Find the equation of motion if the mass is released from the equilibrium position at an upwards velocity of 2m/sec. Assume here that the positive x-direction is oriented downwards.

   (d) Find the first positive time at which the weight returns to the equilibrium position.

5. [10 points] Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 20$ kg/sec. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

6. [10 points] Find the largest interval centered around $x = 0$ in which the initial value problem
$$y'' + \frac{1}{\sqrt{3-x}}y' + \frac{1}{x+1}y = \sin(x), \quad y(0) = 2, y'(0) = 3$$
has a unique solution. Do not try to solve the DE! Use theoretical reasoning instead.