EGR 265, Math Tools for Engineering Problem Solving
October 10, 2012, 50 minutes

Name: . . . .

TEST II

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 2y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = -2.$$

Problem 2 (20 points)

Find the general solution of

$$y'' + 2y' = \sin x.$$

Problem 3 (20 points)

Find the general solution of

$$y'' - 9y = 2e^{3x}.$$

Problem 4 (20 points)

An 4 pound weight stretches an undamped spring by 2 feet.
(a) Working in English units, find the value of the spring constant k in lb/ft and the mass m of the weight in slugs.
(b) Find the equation of motion if the mass is released from the equilibrium position at an upwards velocity of 2 ft/sec. Assume here that the positive x-direction is oriented downwards.
(c) Find the first time at which the weight returns to the equilibrium position. Hint: There is a quick and simple way to do this.

Problem 5 (10 points)

Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 1$ pd-sec/ft. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

(a) Find the largest interval centered about $x = 0$ for which the IVP

$$y'' - \frac{1}{x^2 - 4}y' + 2y = \frac{1}{x^2 - 9}, \quad y(0) = -1, \quad y'(0) = 3$$

has a unique solution.

(b) How many linearly independent solutions does $y'' - \frac{1}{x^2-4}y' + 2y = 0$ have in the interval you found in (a)?

Problem 7 (6 points bonus)

Find the general solution of the 5th order linear DE

$$y^{(5)} = 0.$$

Hint: An educated guess might be the best way to solve this problem.