EGR 265, Math Tools for Engineering Problem Solving
September 12, 2012, 50 minutes

TEST I

Name: ....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Total	

Problem 1 (16 Points)

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $2y'y = \cos x$

(b) $y^4 - xy''' = y' + e^x y$

(c) $y^{(4)} - xy''' = y' + e^x y$

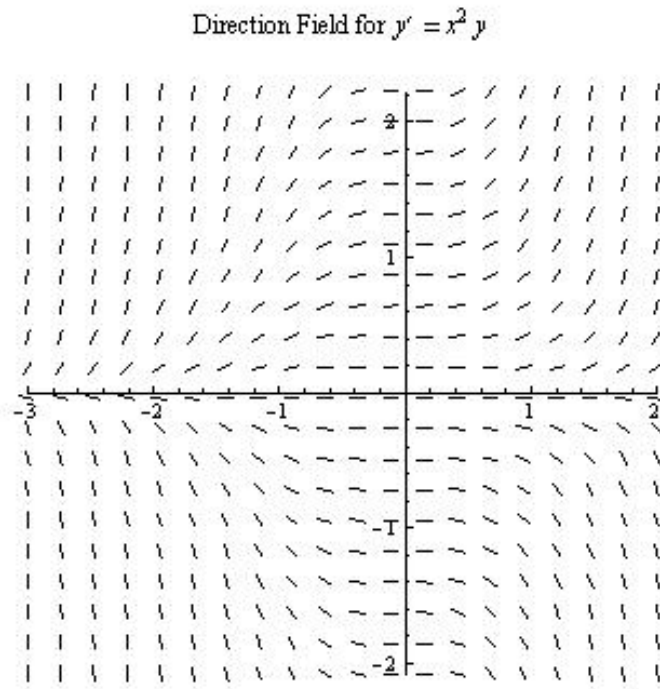(d) $\dfrac{y - \cos x}{y''} = e^x$

Problem 2 (12 Points))

For a given differential equation $x^4 y' - xy^2 + 8x^5 = 0$, solve the following questions.

(a) Find all values of C such that $y = Cx^2$ is a solution of this DE.

(b) Find the unique solution of the DE satisfying the initial condition $y(3) = 36$.

Problem 3 (10 Points)

Below the direction field for $y' = x^2 y$ is given.

Direction Field for $y' = x^2 y$

(a) Sketch the solution $y(x)$ of $y' = x^2 y$ with initial value $y(0) = 1$. (Note: Do not solve the DE!)

(b) From your sketch determine $\lim_{x \to \infty} y(x)$ and $\lim_{x \to -\infty} y(x)$.

(c) Is it possible that two solution curves for $y' = x^2 y$ cross one another in the direction field? Justify!

Problem 4 (16 Points)

Solve the IVP

$$y' + 2x(y-1) = 0, \quad y(0) = 2.$$

Problem 5 (16 Points)

Solve the IVP

$$y' + \frac{1}{x}y = x, \quad y(1) = 1$$

Problem 6 (16 Points)

Solve the IVP

$$2yy' = xe^x, \quad y(0) = -1$$

Problem 7 (14 Points)

A cup of milk is chilled to 35°F in the refrigerator and then taken out into a room of 70°F.
After 5 minutes it has warmed up to 45°F.
Note: Your answers to the questions below will contain natural logarithms which do not need
to be evaluated.

(a) Newton's Law of Cooling can also be used to describe warming processes as in this
problem. Write down the corresponding IVP using an unknown warming rate k.

(b) Solve the IVP and determine k by using information provided in the problem.

(c) When does the temperature of the milk reach 60°F?