EGR 265, Math Tools for Engineering Problem Solving
November 15, 2010, 50 minutes

Name: .....

TEST III

Problem 1 (9+9 points)

(a) Let $f(x, y) = 3x^3 - 2x^2y^2$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x, y) = (x + y) \ln y$ find g_x, g_y and g_{yx}.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = \sqrt{x^2 + y^4}$ find its direction **and** rate of steepest descent at the point $P(1, 1)$.

(b) Find the directional derivative of $h(x, y)$ at $P(1, 1)$ in the direction of the vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of $z = \cos(x + y)$ at the point $(\pi/2, 0, 0)$.

(b) Also, find parametric equations for the normal line to the graph of $z = \cos(x + y)$ at $(\pi/2, 0, 0)$.

Problem 4 (12 points)

Evaluate $\int_C \frac{x}{y}\, ds$, where the curve C is parameterized by $x = t^3/3$, $y = t^4/4$, $1 \leq t \leq 2$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = xe^x \mathbf{i} + 2xy\mathbf{j}$$

along the curve C given by the graph of $y = x^3$, $0 \leq x \leq 1$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = \sin(x)\cos(y)\mathbf{i} - \cos(x)\cos(y)\mathbf{j}$

(b) $F(x, y) = (3x + y^2)\mathbf{i} + 2xy\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = t\sin(\pi t)\mathbf{i} + t\cos(\pi t)\mathbf{j}$, $1/2 \leq t \leq 1$.

Problem 8 (5 points Bonus)

Is it possible that a function $f(x, y)$ has partial derivatives $f_x = x^2 + y^2$ and $f_y = 3xy$? If yes, find such a function $f(x, y)$. If no, give a reason why no such function exists.