Instructor:_____ Name:_____

Exam IV

Calculus I; Fall 2010

Part I

Part I consists of 12 questions, each worth 5 points. Clearly show your work for each of the problems listed.

(1) Let $f(x) = x^4 - 2x^2$. Find all local max/min of $f(x)$. State both x and y coordinates.

(2) Find the absolute max/min of $f(x) = x^3 + 2$ on the interval $[1, 2]$. Give both x and y-coordinates and justify your answer.

(3) Find two positive numbers whose product is 100 and whose sum is minimal. (You must justify your answer.)

(4) Find the number c whose existence is guaranteed by the Mean Value Theorem for the function $y = f(x) = x^4$ on the interval $[1, 2]$.

(5) If $f'(x) = (x-1)^3(x+1)^5$. **Note that you are already given the derivative $\mathbf{f'(x)}$.** Find all critical points, where $f(x)$ is increasing and decreasing, and also find the x-coordinate(s) of all local max/min.

(6) If $f''(x) = (x-1)^3(x+1)^2$ find where $f(x)$ is concave up and where it is concave down. Also find all points of inflection. **Note that you are given $\mathbf{f''(x)}$!**

(7) Find the most general **anti-**derivative of $f(x) = \frac{x^3+2-5\sqrt{x}}{x^5}$.

(8) Find the most general **anti-**derivative of $f(x) = (x+2)(x-1)$.

(9) Find all asymptotes of the function $\frac{x^3+5}{x^2(x-1)(x+2)}$.

(10) If the acceleration is given by $a(t) = 2t + 1$, $v(0) = 1$ and $s(0) = 2$, find $S(2)$.

(11) Find the most general **anti-** derivative of $y = f(x) = \sin(x) + \frac{1}{x^2+1}$

(12) Find the x-coordinate(s) of all local/absolute max/min of the function $y = f(x)$ if $f'(x) = (x-1)^2(x+1)^3$. **Note that you are already given the derivative $\mathbf{f'(x)}$.**

Part II

Part II consists of 3 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) **[10 pts]** Find the absolute max/min of the function $f(x) = (x^3 - 1)^2$ on the interval $[-2, 2]$.

(2) Given the function $f(x) = \frac{(x+1)^2}{1+x^2}$,

(a) **[2 pts]** Find the x and y intercepts of the function.

(b) **[3 pts]** Find all asymptotes.

(c) **[4 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(d) **[2 pts]** Find the local maximum and local minimum value(s) of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).

(g) **[5 pts]** Use the above information to graph the function **on the next page**. Indicate all relevant information in the graph.

Put the graph of Problem 2 on this page.

(3) [**10 pts**] You want to cross a 10 m wide canal to a point on the opposite side which is located 1000 m from the point straight across from you. If you can travel in the water at a speed of 10 m/s and on land at a speed of 1 m/s, determine the path which takes the least time. **You must justify your answer!**