Instructor:____________________ Name:____________________

Final Exam

Calculus I; Fall 2009

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

In 1-4, find y' if:

(1) $y = x^2 \sin(x)$

(2) $y = \frac{\ln(x)}{2x+1}$

(3) $y = (\tan(x))^{30}$

(4) $y = \cos(x^3 + x)$

(5) Find the critical points of $y = f(x) = x(x+1)^3$

(6) Find all local/absolute maxima/minima of the function $y = 2x^4 - x$. Make sure to state both x and y values. (Do **not** simplify these numbers!)

(7) Find all asymptotes of the function $y = \frac{3x^2-1}{x^2-1}$

(8) Find all x-values where $y = x\ln(x)$ is **increasing**

(9) Find the most general form for the **anti-**derivative of $y = x(3x+2)$

(10) Use calculus to find two positive numbers whose product is 4 and whose sum is minimal

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [**10 pts**] Use implicit differentiation to find the derivative if $y^5 = x^2y - x^3$

(2) [**6 pts**] Find the linearization of the function $y = f(x) = \sqrt[4]{x}$ at $x = 16$.

(3) [**4 pts**] Use the linearization in problem 2 to estimate $\sqrt[4]{16.5}$

(4) Given the function $y = f(x) = x^4 - 6x^2$

(a) [**2 pts**] Find the x and y intercepts of the function.

(b) [**2 pts**] Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(c) [**2 pts**] Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(d) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(e) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).

(f) **[5 pts]** Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **absolute maxima/minima**

(5) **[5 pts]** If $y = \frac{(x-1)^2}{(x+1)^3}$ find the absolute maximum and minimum of $f(x)$ on the interval $[0, 5]$. (Include the appropriate y values but do not simplify.)

(6) **[10 pts]** An advertising executive wants to design a can (= cylinder) which is most visible. He decides that this means that the can must have maximal surface area. The can must have a volume of 100 cm^3. Using calculus you must either state the dimensions the can with maximal surface area or show such a can does not exist. [Given a can of radius r and height h its volume $V = \pi r^2 h$ and its surface area $S = 2\pi rh + 2\pi r^2$.]