EGR 265, Math Tools for Engineering Problem Solving
December 8, 2008, 10:45am to 1:15pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$\frac{dy}{dx} = e^{y+2x}, \quad y(0) = 0.$$

Problem 2 (12 points)

(a) Write down the differential equation for the charge $q(t)$ in coulombs at the capacitor in an RC-series circuit including a resistor of R ohms, a capacitor of capacitance C farads and an impressed voltage $E(t)$.
(b) By solving an initial value problem for the differential equation from part (a) determine the charge $q(t)$ in an RC-series circuit if $R = 50$ ohms, $C = 2 \times 10^{-3}$ farads and a constant voltage of $E = 200$ volts is impressed. Assume that after one second of charging the charge on the capacitor is 2 coulombs.
(c) Determine the current $i(t)$ in amperes for the circuit in part (b).

Problem 3 (14 points)

Consider the second order differential equation

$$2y'' - 4y' + 2y = \sin x. \qquad (1)$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 1/2$, $y'(0) = -1$.

Problem 4 (11 points)

A 100-kilogram mass stretches a spring by 10cm. The spring/mass system has no damping and no exterior forcing.
(a) Find a second order differential equation for the position $x(t)$ of the mass relative to its equilibrium position. Use the approximate value $g = 10\,\mathrm{m/s^2}$ for the gravitation constant and assume that the positive x-direction is measured downward from the equilibrium.
(b) Assuming that the mass is released 20cm below the equilibrium position from rest, determine its position $x(t)$.

Problem 5 (11 points)

(a) Find the directional derivative of $f(x, y) = xe^{xy}$ at the point $(1, 0)$ in the direction of the vector $4\mathbf{i} - 3\mathbf{j}$.
(b) Find a unit vector in the direction of steepest decrease of $f(x, y) = xe^{xy}$ at the point $(1, 0)$.

Problem 6 (8 points)

Determine the equation of the tangent plane to the graph of $z = \frac{xy}{x+y}$ at the point $(3, 6, 2)$.

Problem 7 (8 points)

How much work is done by the force field $F(x, y) = x^2y\mathbf{i} - xy\mathbf{j}$ along the curve traced by the vector function $r(t) = t^3\mathbf{i} + t^4\mathbf{j}$, $0 \le t \le 1$?

Problem 8 (12 points)

(a) Show that the force field $F(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$ is conservative.
(b) Find a potential for the force field F from part (a).
(c) Find the work done by the force field F from part (a) along the curve $x(t) = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.

Problem 9 (8 points)

Find the moment of inertia about the y-axis of the lamina given by the region which is bounded by the x-axis and the parabola $y = 4 - x^2$.

Problem 10 (8 points)

Find the double integral of the function $f(x, y) = \frac{1}{\sqrt{x^2+y^2+1}}$ over the washer-shaped region in the xy-plane, centered at the origin, with inner radius 1 and outer radius 2.

Problem 11 (5 points Bonus)

There are two points (x_1, y_1) and (x_2, y_2) at which the function

$$f(x, y) = x^4 - x^2 + y^2 - 2xy - 4x + 4y$$

takes its minimum value. Find these two points!