EGR 265, Math Tools for Engineering Problem Solving
November 20, 2008, 50 minutes

Name (Print last name first): .....

Student ID Number: ....

TEST III

Problem 1 (8+8 points)

(a) Let $f(x, y, z) = x^3y^3z^3$. Find the third order partial derivative f_{xyz}.

(b) Find the directional derivative of $g(x, y) = \sin(xy^2)$ at the point $(\pi, 1)$ in the direction of the vector $3\mathbf{i} - 4\mathbf{j}$.

Problem 1 (10+5 points)

(a) Find a unit vector in the direction of steepest descent of $f(x, y) = \frac{1}{x^2+y}$ at the point $(1, 1)$.

(b) Also, find a unit vector parallel to the level curve of $f(x, y) = \frac{1}{x^2+y}$ through the point $(1, 1)$.

Problem 3 (12+5 points)

(a) Find an equation for the tangent plane to the graph of $z = \ln(x^3 + xy)$ at the point $(1, 0, 0)$.

(b) Also, find parametric equations for the normal line of $z = \ln(x^3 + xy)$ at $(1, 0, 0)$.

Problem 4 (15 points)

Calculate the line integral $\int_C \sqrt{1+4y}\,ds$, where C is the graph of $y = x^2$, $0 \leq x \leq 2$.

Problem 5 (15 points)

Find the work done by the force field

$$F(x,y) = e^{y^2}\mathbf{i} + xy\mathbf{j}$$

along the curve parametrized by $x = t^4$, $y = t^2$, $0 \leq t \leq 1$.

Problem 6 (16+6 points)

(a) Is the force field

$$F(x,y) = (1 + 2xy^2)\mathbf{i} + (3y^2 + 2yx^2)\mathbf{j}$$

conservative? If yes, find a potential for F.

(b) For the force field $F(x,y)$ from part (a) find the work done by F along the curve parametrized by

$$x = 2\cos t, \quad y = 2\sin t, \quad 0 \le t \le \pi/2.$$