EGR 265, Math Tools for Engineering Problem Solving
September 15, 2008, 50 minutes

Name (Print last name first): . . . .

 Student ID Number: . . . .

$$\boxed{\boxed{\textbf{TEST I}}}$$

<u>Problem 1</u>

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P)

   (a) $yy'' = \cos x$

   (b) $y^4 - y''' = e^x y$

   (c) $\dfrac{y - \cos x}{y'} = e^x$

<u>Problem 2</u>

Which of the functions $y = 0$, $y = 2$, $y = 2x$ and $y = 2x^2$ is a solution of the following DEs? (5P+5P+5P)
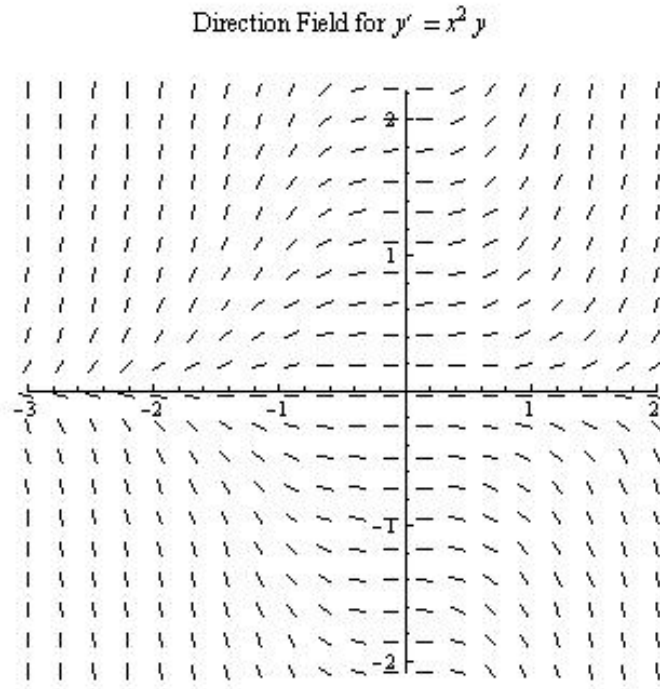
   (a) $xy' = y$

   (b) $y'' + 9y = 18$

   (c) $xy'' - y' = 0$

Note: The DEs (a), (b) and (c) may have more than one solution. Behind each of them list all correct solutions.

Problem 3

Below the direction field for $y' = x^2 y$ is given.

Direction Field for $y' = x^2 y$

(a) Sketch the solution $y(x)$ of $y' = x^2 y$ with initial value $y(0) = 1$. (6P)

(b) From your sketch determine $\lim_{x \to \infty} y(x)$ and $\lim_{x \to -\infty} y(x)$. (4P)

(c) Find the solution of the IVP $y' = x^2 y$, $y(0) = 1$ explicitly. (12P)

Problem 4

Solve the IVP (17P)

$$\frac{y'}{x} + 2y = 1, \quad y(0) = 1$$

Problem 5

The percentage $p(t)$ of a population which uses cell phones is described by the logistic DE $p' = p(1 - p)$, where t is measured in years. Initially, 10 percent of the population use cell phones, i.e. $p(0) = 0.1$. Solve the DE to find a formula which gives $p(t)$ for arbitrary times t. Note that, as $p(t)$ is a percentage, we will always have $0 < p(t) < 1$. (17P)

Problem 6

In the electrical circuit below one has $R = 1000$ ohms, $C = 0.001$ farads and a constant electromotive force of $E(t) = 50$ volts.
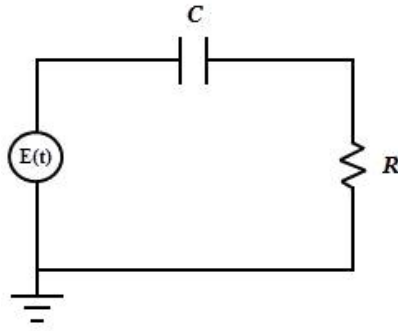
(a) Write down the DE for the charge $q(t)$ and solve it to find a closed form solution for $q(t)$ in Coulombs given that $q(0) = 0$. (13P)

(b) Find a formula for the current $i(t)$ in amperes. (4P)