FALL 2008 — MA 227 — FINAL

Name: ______________________________

1. Part I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Compute the cross product of the vectors $\langle 1, 1, 0\rangle$ and $\langle 2, 1, -2\rangle$.

____

(2) Find the gradient of the function $f(x, y, z) = x^2 + 2y^2 + e^{-z}$.

____

(3) Find the directional derivative of the function $h(x, y) = xy^{1/2}$ at the point $(1, 1)$ into the direction of the point $(2, 2)$.

____

(4) Find a parametrization for the curve which intersects the cylinder $x^2 + y^2 = 1$ and the plane $z = x + 2$.

____

(5) Find a parametrization of the plane spanned by the vectors $\langle 1, 2, 1\rangle$, and $\langle 0, 1, 0\rangle$ and passing through the point $(0, 0, 1)$.

____

(6) Find the linearization $L(x, y)$ of $f(x, y) = ye^x$ at the point $(0, 2)$.

(7) Evaluate $\int_S 3dS$ where S is the unit circle with radius 1.

(8) Change the order of integration in the iterated integral:

$\int_0^1 (\int_y^1 f(x, y)\, dx)\, dy$.

(9) Compute $\operatorname{div} \mathbf{F}$ when $\mathbf{F}(x, y, z) = \langle x^2, y^2, z^2 \rangle$.

(10) Compute $\operatorname{curl} \mathbf{F}$ when $\mathbf{F}(x, y, z) = \langle xy, \cos(xy), 0 \rangle$.

2. Part II

There are 6 problems in Part II, each worth 10 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Find an equation of the plane passing through $A(0, 1, -0)$, $B(2, 1, 1)$, and $C(0, -1, 1)$. What is the angle between this plane and the xy-plane?

(2) Find and classify the critical of the function

$$f(x, y) = y^3 + yx^2 + 3y^2 + x^2$$

points. In other words find the local maximum and minimum values and saddle points.

(3) Evaluate the integral $\iiint_E (x^2 + y^2)dV$, where E is the cone which lies between $z = 0$ and $z = 1 - \sqrt{x^2 + y^2}$.

(4) Let C be the circular arc from $(1,0)$ to $(-1,0)$. Evaluate $\int_C x\,dx + y\,dy$ by using the fundamental Theorem of line integrals as well as by direct integration.

(5) A surface S is given parametrically by $\mathbf{r}(u, v) = \langle u, v, 1 - u^2 - v^2 \rangle$ where $u^2 + v^2 \leq 1$. Evaluate the area of the surface S.

(6) Let $\mathbf{F}(x, y, z) = \langle x, y, 0\rangle$ and S the cylinder $x^2 + y^2 = 1$ with the lids $z = 0$ and $z = 2$. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{dS}$ in two ways. By direct integration as well as by using Gauss' Theorem (or the divergence theorem), which states that

$$\iint_S \mathbf{F} \cdot \mathbf{dS} = \iiint_E \operatorname{div} \mathbf{F} \, dV$$

where denotes the solid of the corresponding cylinder.