MA 126 - CALCULUS II December 07, 2007

Name (Print last name first):. . . . . .

Signature: . . . . . .

Student ID Number (last four digits): . . . .

Section: . . . . . Instructor Name: . . . . . .

FINAL EXAM

Closed Book. No calculators are permitted.

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 8 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the exam. No partial credit is awarded for this part of the exam! CHECK YOUR ANSWERS!

Question 1

Find the **center** and the **radius** of the sphere $x^2 + y^2 + z^2 + 2x - 8y + 6z + 1 = 0$.

Answer: . . . . . .

Question 2

Find a **vector** that is **orthogonal** to both of the vectors $< 1, -1, 2 >$ and $< -1, 0, 3 >$.

Answer: . . . . . .

Question 3

Find the **parametric equations** of the line through the point $(3, 2, -5)$ and parallel to the vector $< -1, 1, 6 >$.

Answer: .....

Question 4

Evaluate the **indefinite** integral

$$\int \frac{1}{x} \cos(\ln x)\, dx.$$

Answer: .....

Question 5

Find the **derivative** $f'(x)$ if

$$f(x) = \int_3^x e^{t^2} dt.$$

Answer: .....

Question 6

Evaluate the **definite** integral

$$\int_0^{\pi} x \sin x \; dx.$$

(Your answer must be a number!)

Answer: .....

Question 7

Find the **area** of the region **bounded** by the parabola $y = 4 - x^2$ and the x-axis.
(Your answer must be a number!)

Answer: .....

Question 8

Determine whether the **series** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^4}}$ diverges, converges conditionally, or converges absolutely. (Only one of these three choices will be accepted as an answer!)

Answer: .....

PART II - Problem Solving skills

Each problem is worth 12 points.

Part II consists of 5 problems. You must show your work on this part of the exam to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit. CIRCLE YOUR ANSWER!

Problem 1

The velocity function (in miles per second) of an object moving along a line is given by

$$v(t) = 2t - 4, \qquad 0 \leq t \leq 4.$$

(a) Find the **displacement** (in miles) of the object during the given time interval.

(b) Find the **distance traveled** (in miles) by the object during the given time interval.

Problem 2

This problem has two separate questions. (Answer all questions!)

(a) Find the **parametric equations** for the **line** passing through the points $(2, 0, 1)$ and $(-1, 3, 2)$, and then determine the coordinates of **points** where the **line intersects** the coordinate planes, i.e., the xy-plane, the xz-plane, and the yz-plane.

(b) Consider two planes given by the equations $x + y + z = 1$ and $x - y = 1$, respectively.

(b_1) Find the **symmetric equations** for the **line of intersection** of the two planes.

(b_2) Find an **equation of the plane** containing the point $(0, 1, 1)$ and the line of intersection obtained in (b_1) above.

[Hint: Some info from your calculations in (b_1) might prove useful here, or alternatively, you might use two points on the line obtained in (b_1) and the point given here to proceed!]

Problem 3

Evaluate the following integrals and general antiderivatives (clearly show the techniques of integration you use):

(a)

$$\int x \ln x \, dx$$

(b)

$$\int \tan^3 x \, dx.$$

[Hint: Recall that $\tan x = \dfrac{\sin x}{\cos x}$, and that $\sin^2 x = 1 - \cos^2 x$.]

(c)

$$\int_1^3 \frac{2}{x^3 + 2x} \, dx$$

(Your answer must be a number!)

Problem 4

(a) Find the **length of the arc** traced by the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}$$

when $1 \leq x \leq 2$. (Simplify your answer!)

(b) Find the **volume** of the solid generated by rotating the region **bounded** by the parabolas $y = x^2 - 1$ and $y = 1 - x^2$ about the x-axis.

Problem 5

This problem has two separate questions. (Answer all the questions!)

(a) **Test the following series for convergence.** In order to receive credit, you must state the test that you are using and give the reasoning for your conclusion.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n}$$

(b) Consider the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^{n-1}}{e^{n-1}}.$$

(b_1) Find the **values** of x for which **the series converges**. (Write your answer in interval notation!)

(b_2) Find **the sum of the series** for those values of x. (You must simplify your answer!)

SCRATCH PAPER

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