MA 126-6A, CALCULUS II

September 24, 2007

Name (Print last name first): .....

Student ID Number (last four digits): ....

TEST I

Closed book test – No calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find an equation of a sphere if one of its diameters has endpoints $(0, -1, 3)$ and $(1, 1, 5)$.

Answer: .....

Question 2

Find the angle between the vectors $\mathbf{u} = \mathbf{i} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j}$. (Express your answer in radians or in degrees.)

Answer: .....

Question 3

For what numerical values of b are the vectors $\langle -4, b^2, -5\rangle$ and $\langle -4b, b, 5b\rangle$ orthogonal?

Answer: .....

Question 4

Find a vector which is orthogonal to both $\langle 2, 0, -3\rangle$ and $\langle -1, 1, -2\rangle$.

Answer: .....

Question 5

Find parametric equations for the line through the points $(3, 2, -5)$ and $(2, 3, 1)$.

Answer: .....

Question 6

Find an equation of the plane through the points $(0, 2, 1)$, $(3, 0, 1)$, and $(5, 2, 0)$.

Answer: .....

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) A woman exerts a horizontal force of 17 lb on a stroller as she pushes it up a ramp that is 20 ft long and inclined at an angle of 60° above the horizontal. Find the work done on the stroller.

(b) Right before touchdown an astronaut is steering the space shuttle in the direction N30°E (that is to say, 30° East of due North) at an airspeed (speed in still air) of 60 mph, and the wind is blowing from the direction N30°W (that is to say, 30° West of due North) at a speed of 60 mph.

(1) Find the true course of the space shuttle. (Express your answer as an angle with respect to specific direction(s) North, South, East, or West.)

(2) Find the 'ground' speed of the shuttle. (That is, the magnitude of the resultant-velocity!)

Problem 2

(a) Find the area of the triangle with vertices $P(5,5,5)$, $Q(5,6,5)$, and $R(5,5,6)$.

(b) Find the volume of the box determined by the vectors $\mathbf{a} = \langle 6,3,-1\rangle$, $\mathbf{b} = \langle 0,1,2\rangle$, and $\mathbf{c} = \langle 4,-2,5\rangle$.

Problem 3

Consider the two lines given by the parametric equations

$$\begin{aligned} \mathcal{L}_1: &\quad x = 1 + 2t, \qquad y = 3t, \quad z = 2 - t \\ \mathcal{L}_2: &\quad x = -1 + s, \quad y = 4 + s, \quad z = 1 + 3s \end{aligned}$$

(a) Determine whether these lines are parallel. (Justify your answer!)

(b) Determine whether these lines intersect. If they do, find the point of intersection. Otherwise justify your answer.

(c) Determine whether these lines are skewed. (Justify your answer!)

Problem 4

Find parametric and symmetric equations of the line of intersection of the planes $x+y-z=2$ and $3x-4y+5z=6$. (Clearly indicate which ones are parametric equations and which ones are symmetric equations.)

Problem 5

An astroid is traveling in space along the space-curve

$$\mathbf{r}(t) = \left\langle \frac{1}{t+1},\ e^{-t},\ \frac{\sin t}{t} \right\rangle$$

when the time $t > 0$.

(a) At what point in space is the astroid located when the time $t = 0$.
[Hint: L'Hospital's rule might prove useful!]

(b) Evaluate $\lim_{t\to\infty} \mathbf{r}(t)$; that is, find where the astroid is headed as time goes by.

(c) Determine the velocity (vector) of the astroid at each time t.

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