FALL 2006 — MA 227-8B — TEST 2

Name: ____

1. Part I

There are 4 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Describe or sketch the domain of the function $f(x, y) = \ln(4 - x^2 - y^2)$.

Answer: $\{(x, y) \colon x^2 + y^2 < 4\}$, a disk (without boundary) of radius 2 centered on $(0, 0)$.

(2) Find the first partial derivatives of $f(x, y) = x^2y - \frac{x}{y}$.

Answer: $f_x = 2xy - 1/y$, $f_y = x^2 + x/y^2$.

(3) Find the linearization $L(x, y)$ of $f(x, y) = x \cos y$ at the point $\langle 2, \pi/3\rangle$.

Answer: $L(x, y) = \pi/\sqrt{3} + \frac{1}{2}x - \sqrt{3}y$.

(4) Find the gradient of $f(x, y, z) = z^2 \sin(x - y)$.

Answer: $\langle z^2 \cos(x - y), -z^2 \cos(x - y), 2z \sin(x - y)\rangle$.

2. Part II

There are 3 problems in Part II, each worth 8 points. On Part II problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Define the term "critical point of f" and explain why they are important for finding maxima and minima. Then find and classify the critical points of $f(x, y) = 2x^3 + xy^2 + 6x^2 + y^2$.

Answer: $(0, 0)$ is a local minimum, $(0, -2)$ is a local maximum, $(-1, \sqrt{6})$ and $(-1, -\sqrt{6})$ are saddle points.

(2) Use Lagrange multipliers to find the minimal and maximal values of $f(x, y) = x + 4y$ on the circle $x^2 + y^2 = 1$. Where do they occur?

Answer: $f = \sqrt{17}$ at $(1/\sqrt{17}, 4/\sqrt{17})$ is the maximum; $f = -\sqrt{17}$ at $(-1/\sqrt{17}, -4/\sqrt{17})$ is the minimum.

(3) Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = xz^2 - 4y\sqrt{x}.$$

(a) Find the vector $\overrightarrow{PQ}$ which points from $P(4, 1, -1)$ to $Q(2, 3, 0)$ as well as the unit vector pointing in the same direction.

Answer: $\overrightarrow{PQ} = \langle -2, 2, 1\rangle$, the unit vector $\mathbf{u} = \langle -2/3, 2/3, 1/3\rangle$.

(b) Find the rate of change of the potential at the point P in the direction towards the point Q.

Answer: gradient is $\nabla f(P) = \langle 0, -8, -8\rangle$, the rate of change is $\nabla f(P) \cdot \mathbf{u} = -8$.

(c) In which direction does V change most rapidly at P?

Answer: in the direction of the gradient, $\langle 0, -8, -8\rangle$.

(d) What is the maximum rate of change of V at P?

Answer: $|\nabla f(P)| = 8\sqrt{2}$.